A Unifying Approach and Interface for Spline-based Snakes

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Active Contours — Snakes

Ordered collection of points with an internal energy and is guided by

1. Image Energy

   Guides the snake to the image features

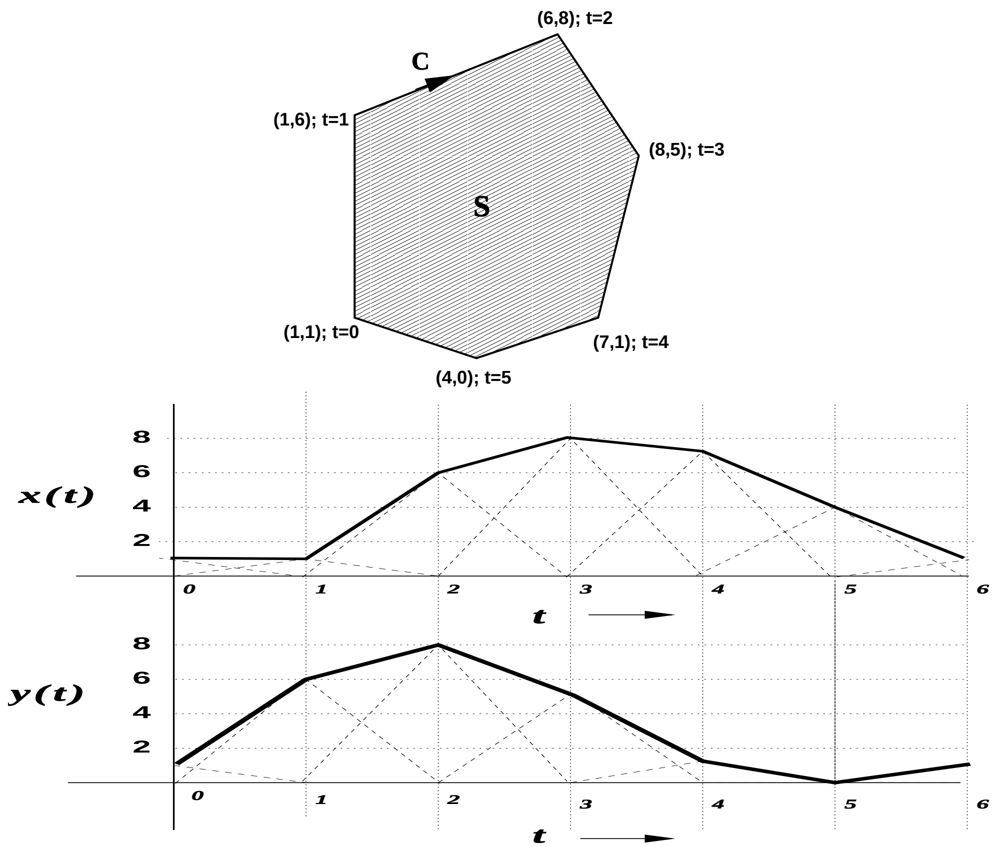
2. External constraint energy

   Constrain the snake using the apriori knowledge

Piecewise constant representation $\longrightarrow$ Need for explicit Internal Energy

$\leadsto$ Smoother basis functions

Scaling function representation of curves



$$\mathrm{r}(t) = \sum_{k=0}^{M-1} \mathrm{b}_k \, \varphi_p(t-k); \qquad \mathrm{r}(t) = (x(t), y(t))$$

where

$$\varphi_p(t) = \sum_{k=-\infty}^{\infty} \varphi(t - kM)$$

Cubic B-Spline Representation

$$\varphi(t) = \beta^3(t),$$

$$\widehat{\beta}^n(\omega) = \left(\frac{1 - e^{-j\omega}}{j\omega}\right)^{n+1}; \quad n = 3$$

- Minimum Curvature Interpolation

  Cubic B-Spline interpolation $\longrightarrow$ Minimum curvature curve (if described in the curvilinear abscissa)

  $\Rightarrow$ Simpler optimization scheme

- Good Approximation Property

- Efficient signal processing algorithms

Generic form of Image Energy

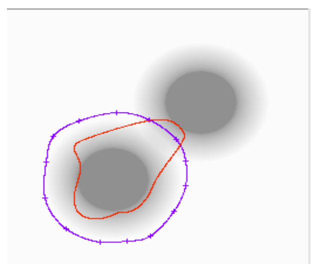
$$E^c_{\text{image}} = \int_{\mathcal{S}} g(x,y)dxdy,$$

where $g = Tf$

Gradient-based Image Energy

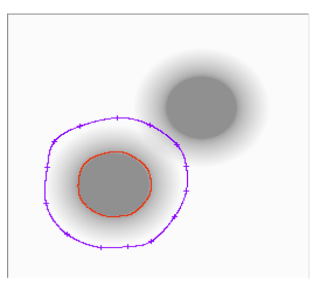
Widely used scheme

$$E^g_{\text{image}} = \oint_{\mathcal{C}} |\nabla f(\mathbf{r})|^2 dr$$



Alternative method

$$E^g_{\text{image}} = \oint_{\mathcal{C}} \mathbf{k} \cdot (\nabla f(\mathbf{r}) \times d\mathbf{r}) = \int_{\mathcal{S}} \underbrace{-\nabla^2 f(\mathbf{s})}_{T_g(f)} d\mathcal{S}$$



Advantages and Disadvantages of Gradient scheme

- Precise
- Lack of convergence away from the contour
- Sensitive to noise
- Difficulty to move into region concavities

Region-based Image Energy

$$E^r_{\text{region}} = \int_{\mathcal{S}} \log\left(P\left(f(\mathbf{s}) \,|\, \mathbf{s} \in \mathcal{S}\right)\right) ds + \int_{\mathcal{S}'} \log\left(P\left(f(\mathbf{s}) \,|\, \mathbf{s} \in \mathcal{S}'\right)\right) ds$$

Removing a constant term,

$$C = \int_{\mathcal{S} \cup \mathcal{S}'} \log\left(P\left(f(\mathbf{s}) \,|\, \mathbf{s} \in \mathcal{S}'\right)\right) ds$$

we get

$$E^r_{\text{region}} = \int_{\mathcal{S}} \underbrace{\log\left(\frac{P\left(f(\mathbf{r}) | \mathbf{s} \in \mathcal{S}\right)}{P\left(f(\mathbf{s}) | \mathbf{s} \in \mathcal{S}'\right)}\right)}_{T_r(f)} ds$$

Advantages and Disadvantages of Region scheme

- Converges away from the contour
- Less sensitive to noise
- Can move into boundary concavities
- Less precise as compared to Gradient scheme

⇝ Combine the two schemes

Combined Image Energy

$$g = \underbrace{\left(\alpha T_g + (1-\alpha)\, T_r\right)}_{T_c} f; \qquad 0 \leq \alpha \leq 1$$

Can possess the good properties of both schemes

Justification for edge enhancement in segmentation schemes

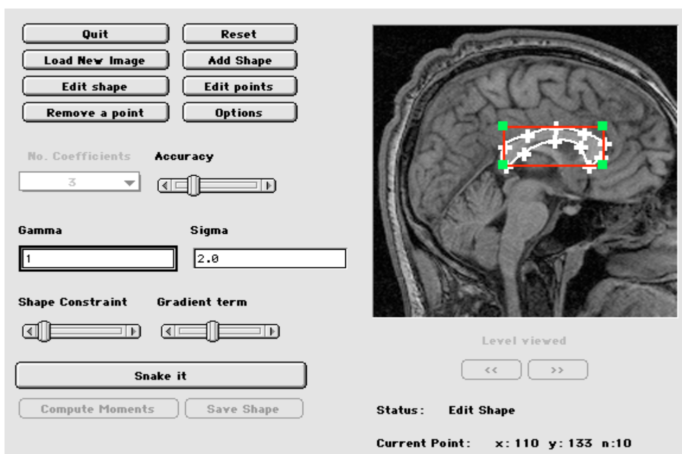
External Constraint Energy

Making use of the apriori knowledge of the shape

Shape Input Mode

- Object is assumed to have roughly the same shape

- User transforms a shape template to fit the object



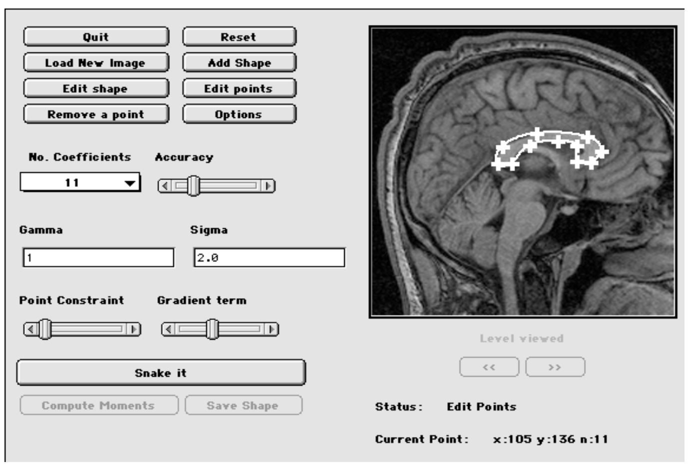
$$E_{\text{constraint}} = \min_{\mathbf{A},\mathbf{b}} \frac{1}{\sigma^2} \sum_{k=0}^{N-1} \left| \mathbf{c}_k - \mathbf{A}\mathbf{c}_{\text{ref},k} - \mathbf{b} \right|^2$$

Valid as B-Spline representation is affine-invariant and is a Riesz basis

- Two step optimization strategy
- Cheaper alternative to eigen shape algorithm

Point Input Mode

- Spline with a specified number of knots are fit
- Reparametrized to the curvilinear abscissa



$$E_{\text{constraint}} = \frac{1}{\sigma^2} \sum_{k=0}^{N-1} \left| \mathbf{r}_k - \mathbf{r}_{\text{init},k} \right|^2 ; \qquad \mathbf{r}_k = \mathbf{r}\left(t_k\right)$$

Optimization Scheme

- Conjugate Gradients Optimization scheme
- Multiresolution framework $\rightarrow$ robust, fast, less sensitive to initial conditions
- Green's theorem $\rightarrow$ efficient evaluation of surface integrals

$\Rightarrow$ Possibility of a looping snake

Can be detected by checking

$$\theta_{\text{total}} = \int_0^M \frac{d\theta(t)}{dt} dt = \int_0^M \frac{x'(t)y''(t) - y'(t)x''(t)}{x'(t)^2 + y'(t)^2} dt$$

The snake is looping if $\theta_{\text{total}} \neq 2\pi$

- Gaussian p.d.f → parameters estimated at every line minimization
- Sign of area → curve direction
- Exact computation of the moments

Conclusions

- Cubic B-Spline representation $\rightarrow$ Natural choice
- Generic form of external energy
- New Gradient-based energy
- New Shape constraint $\rightarrow$ cheaper alternative to Taylor and Cootes