High-Quality Wavelet Splatting for Volume Rendering

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Abstract
We extended Gross’s et al. [1,3] method of volume wavelet rendering by computing splats via least squares approximation. The method decomposes the volume data into a wavelet pyramid representation in the B-spline domain. The splats of the basis functions are projected onto a multiresolution grid. The approximation error on the grid is derived, applying [2], as a function of the sampling step $h$. Choosing at each step the appropriate wavelet space and spatial resolution produces the smallest possible filters and reduces computation. Our approach ensures a maximal image quality when rendering at low resolution.

keywords: volume rendering, wavelet splat, approximation theory, multiresolution

Motivation
Volume rendering is a useful technique for biomedical and geological data visualisation. It is computationally expensive and consumes a lot of memory and bandwidth. Fast implementations rely on hardware rendering and rather coarse approximations. A user would like to browse interactively around and through a volume to get a better imagination of its three-dimensionality. One might accept the compromise of lower image resolution or quality for the sake of fast feedback. Then once a view point of the volume has been chosen, the quality improves. Especially physicians need a high visualisation quality to get reliable diagnosis.

Principle
The volume rendering integral in its simplest form is a parallel projection, and the decomposition of the volume into basis functions is linear. Therefore the X-ray image can be re-composed by the parallel projections of the volume basis functions, the wavelet splats, which can be calculated using the Fourier slice theorem. The volume is rendered at multiple resolutions on an approximation grid. The sampling step $h$ adapts to the scale of the basis function; $h$ can be chosen such that the approximation error is below a desired threshold. The discrete wavelet splat is pre-calculated by convolving the wavelet splat with all possible shifts of the grid basis function. Then for the purpose of visualisation, the approximated X-ray transform is expanded in B-spline space to the full resolution. The adapted resolution and the pre-stored wavelet splat make a fast algorithm possible, whereas the image quality can be tuned with the sampling step size $h$.

References