SIGNAL PROCESSING IV: Theories and Applications
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Optimal Speed Measurement by Optical Flow Using Line Image Sequences

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This paper presents an error analysis of optical flow computation using line image sequences and proposes a solution to optimal average speed measurement.

1. INTRODUCTION

1.1. Optical flow

The optical flow estimation is based on the intensity gradient of image sequences. It has been investigated by many workers, e. g. [3], [4], [5], [8], [9], [11]. These mainly show how to compute a multidimensional flow field, that not only gives velocity magnitude, but also its direction. Such a task has the final goal of providing important clues on the three dimensional world [1], [7]. Unfortunately, this cannot be done without assuming some a priori knowledge because of the aperture problem, that basically specifies that motion can locally only be measured along the intensity gradient.

1.2. Line image sequences

We restrict ourselves to the simpliest of these assumptions, namely that the motion is along a straight line of known direction. In the practical configuration, we use a line image camera that produces a space-time two-dimensional image [2]. A pair of such images taken on neighbouring parallel lines can be used to fully characterize the velocity field. The configuration used to obtain them is showed in figure 1 of reference [10].

1.3. Previous work

We developed the method of flow estimation in a previous work [10] and established that, being local, the measurements obtained are quite noisy.We showed that for each measured velocity V_x that is non-zero we can also compute a relative sensitivity S_r, which behaves much like the inverse of a confidence. We selected a threshold S_{r0} above which the velocity measurements were considered as no more reliable and were rejected. With this method, we improved noticeably the performance of speed measurements over a set Ω of points. Now we intend to introduce an automatic thresholding in the noise reduction scheme. We want to find the threshold which minimizes the velocity estimation error.

2. OPTIMIZATION BASIS

2.1. Hypothesis

First, we introduce some assumptions:

2.1.1. Some segmentation process has provided us with a set Ω of points that covers part of the image and corresponds to a single moving object, which has a constant real speed v_r.

2.1.2. We assume that every point in Ω is sorted with respect to its relative sensitivity, yielding a set of partitions $\omega(S_{r0})$. For each threshold S_{r0}, we know the number of points $CARD(\omega(S_{r0}))$ present in the thresholded subset $\omega(S_{r0})$, as well as its estimated standard deviation $\hat{\sigma}(S_{r0})$ and estimated mean speed $\hat{v}_x(S_{r0})$.

2.1.3. We also assume that the velocity distributions relative to the subsets $\omega(S_{r0})$ do exist, and have all the same average, equal to the correct value v_r and estimated by $\hat{v}_x(S_{r0})$. A further hypothesis is that they are Gaussian, with unknown standard deviation, possibly different for each S_{r0}.

2.2. Confidence interval

Then we remember [6] that over some finite set of measurements $\omega(S_{r0})$, whose distribution is Gaussian, there exists an interval $L(n(S_{r0}), \hat{v}_x(S_{r0}), \hat{\sigma}(S_{r0}), \gamma)$ dependent from the cardinality $n(S_{r0}) = CARD(\omega(S_{r0}))$ and standard deviation $\hat{\sigma}(S_{r0})$, which covers with probability γ the real average v_r of the set $\omega(S_{r0})$. This interval is symmetric with regard to the estimated average $\hat{v}_x(S_{r0})$:

(1) $$L(n, \hat{v}_x, \hat{\sigma}, \gamma) = \left[\hat{v}_x - C_\gamma \cdot \frac{\hat{\sigma}}{\sqrt{n}},\ \hat{v}_x + C_\gamma \cdot \frac{\hat{\sigma}}{\sqrt{n}} \right]$$

Where $C\gamma$ is a coefficient related to γ.

2.3. Criterion

The criterion we will use is to minimize the length of this interval, with respect to the threshold S_{r0}. It is obvious from the above equation that we search to minimize the ratio:

(2) $$r(S_{r0}) = \frac{\hat{\sigma}(S_{r0})}{\sqrt{n(S_{r0})}}$$

This ratio is γ-independant. The S_{r0} found is termed the optimal relative sensitivity and named S_{ropt}.

3. ALGORITHM

3.1. Definitions

Consider a set of two line image sequences, where X is the coordinate relative to the line position, Y is the coordinate along the line and T is the time axis:

(3) $i_1 = f(X_1, Y, T)$
(4) $i_2 = f(X_2, Y, T)$

Choose four points from these images:

(5) $A = f(X_1, Y, T_1)$ (7) $C = f(X_2, Y, T_1)$
(6) $B = f(X_1, Y, T_2)$ (8) $D = f(X_2, Y, T_2)$

Let $\Delta X = X_2 - X_1$ be the distance between the two acquisition lines, and $\Delta T = T_2 - T_1$ be the time interval between two line shots. The velocity flow field and its relative sensitivity are then [10]:

(9) $$V_x(A, B, C, D) = -\frac{\Delta X}{\Delta T} \cdot \frac{B + D - A - C}{C + D - A - B}$$

(10) $$S_r(A, B, C, D) = 4 \cdot \frac{|B - C| + |D - A|}{|(B - C)^2 - (D - A)^2|}$$

We define a collection of subsets $\omega(S_{r0})$ such that:

(11) $$\omega(S_{r0}) = \{ (A, B, C, D) \mid S_r(A, B, C, D) < S_{r0} \}$$

It is important to notice here that due to the discrete nature of observable intensities (A, B, C, D), the set of possible S_r is a finite one, and such is the set of meaningfull S_{r0}.

We then define some operations within a subset, which yield respectively the averaging and the standard-deviation estimations:

(12) $$\hat{v}_x(S_{r0}) = \frac{1}{CARD(\omega(S_{r0}))} \cdot \sum_{(A, B, C, D) \in \omega(S_{r0})} V_x(A, B, C, D)$$

(13) $$\hat{\sigma}(S_{r0}) = \sqrt{\frac{\sum_{(A, B, C, D) \in \omega(S_{r0})} (V_x(A, B, C, D) - \hat{v}_x(S_{r0}))^2}{CARD(\omega(S_{r0})) - 1}}$$

3.2. Computation

i) Get images $i_1(Y, T)$ and $i_2(Y, T)$.
ii) Determine Ω by some segmentation process.
iii) Compute $V_x(Y, T)$ and $S_r(Y, T)$.
iv) Sort points of Ω with regard to $S_r(Y, T)$ in order to get $\omega(S_{r0})$.
v) For each $\omega(S_{r0})$, obtain $\hat{\sigma}(S_{r0})$, $CARD(\omega(S_{r0}))$, $\hat{v}_x(S_{r0})$, $r(S_{r0})$.
vi) Search $S_{r0} = S_{ropt}$ which minimizes the ratio $r(S_{r0})$.
vii) When found, $\hat{v}_x(S_{ropt})$ is the optimal speed.

4. EXPERIMENT

The method described above has been tested with succes using line image sequences of car models. A threefold increase in precision could be obtained with respect to raw measurements.

Figure 1: See reference [10].

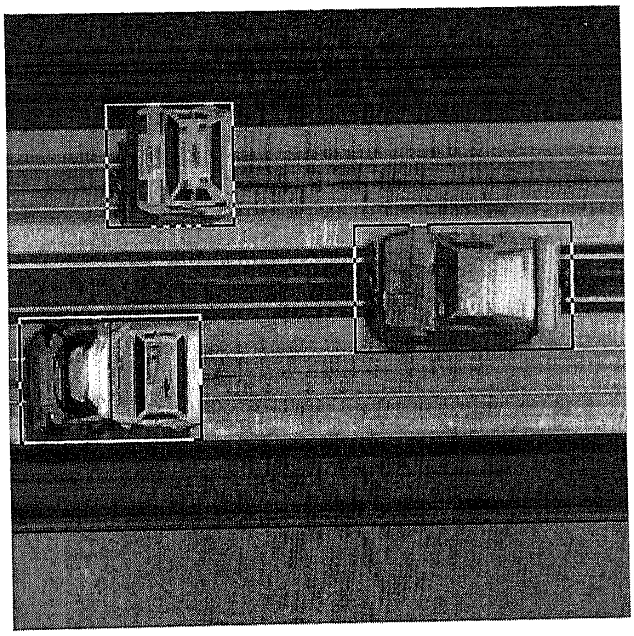

Figure 2 shows an example of a line image sequence, in which one can see a swift pair of cars tied together (lower left), a single swift car (upper left) and a slower car (middle right). Superimposed over the image, a frame encloses each vehicle; it represents the outer limit of Ω, and was obtained by some segmentation process we don't describe here. Note that the shape of these frames, although not optimal, is well suited to the models. This image i_1 was obtained from the first camera seen in figure 1 of reference [10]; the second one produced an image slightly different, but visually undistinguishable from the first one.

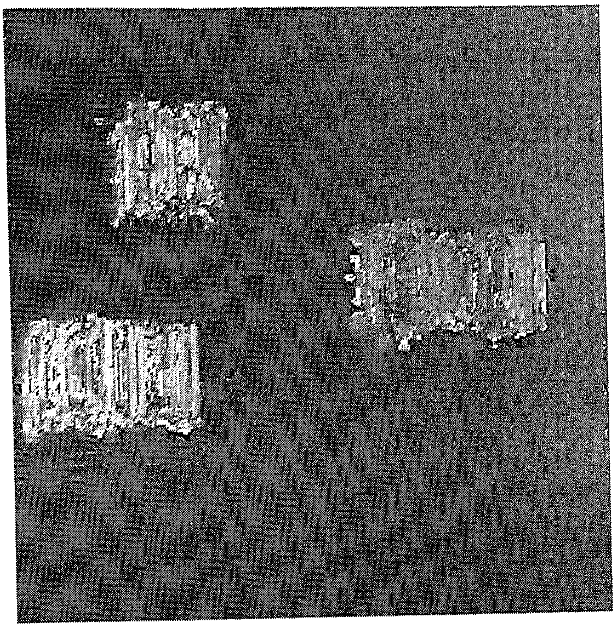

Figure 3 shows the optical flow field computed. One can see that the measured velocities are quiet noisy, and cannot be used directly for segmentation purposes, for example by a velocity thresholding method. One can also see some coarse speed differences between the models: the swiftiest (lighter) are the left ones, the slowest (darker) the right one.

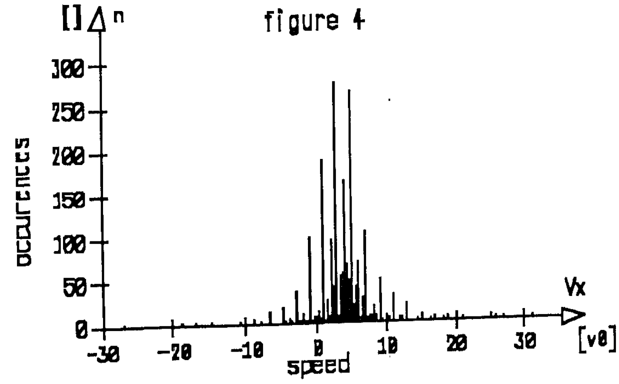

Figure 4 shows the distribution of velocities among the set Ω corresponding to the pair of cars. Each vertical bar represents a discrete speed value, since (A, B, C, D) in equations (5) through (8) are discrete-valued. The number of points corresponding to that particular speed is displayed by the length of the bar. One can see that the distribution is roughly Gaussian.

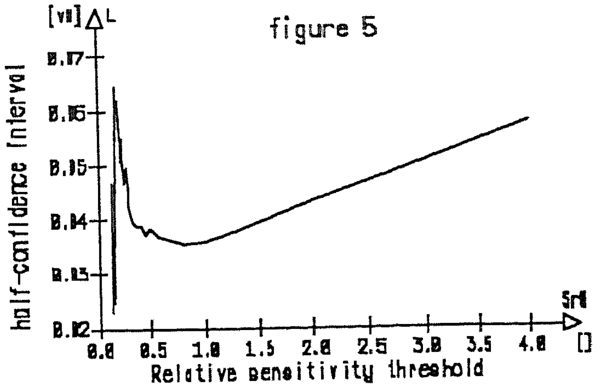

Figure 5 shows the optimization curve $r(S_{r0})$ for the pair of cars. One can see an absolute minimum, which exists for very low relative sensitivity threshold. This minimum has indeed no meaning, since too few points participate to the averaging process. This situation arises whenever some very few points has a low S_{r0} and casually exhibit a very similar V_x. In this case, $\hat{\sigma}(S_{r0})$ will be so small that the minimum of $L(S_{r0}, \gamma)$ is obtained, even if $CARD(\omega(S_{r0}))/CARD(\Omega(S_{r0}))$ is negligible. To avoid this trap, we have designed a minimum fraction before which no optimization takes place; this parameter was set to 20%. Therefore, only significant (for common sense) results are retained. In the case of figure 5, a clear significant minimum does exist, although not absolute.

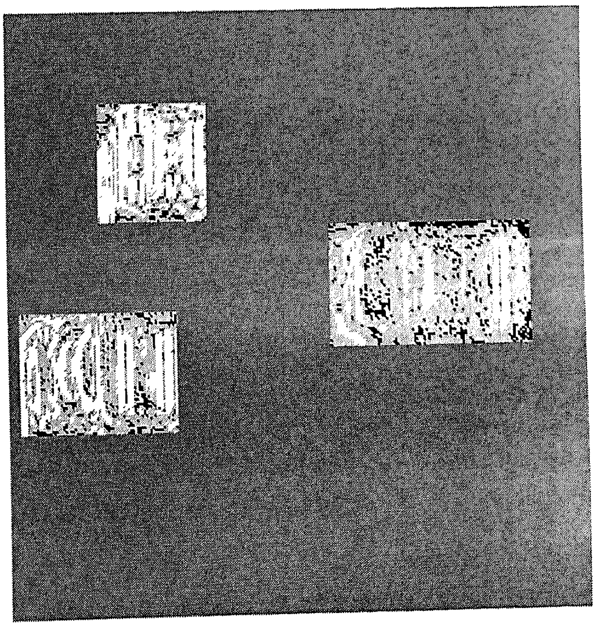

Figure 6 shows the result of thresholding with S_{ropt}. In this image, white codes the points with a sufficient confidence to be used as input for the computation of the optimal speed. It is interesting to note that for each set Ω, a different threshold S_{ropt} had to be used, yielding a different ratio $CARD(\omega(S_{ropt}))/CARD(\Omega(S_{ropt}))$.

Object	Ω $\hat{v}$ [v_0]	$\hat{\sigma}$ [v_0]	$\omega(S_{ropt})$ $\hat{v}_{opt}$ [v_0]	$\hat{\sigma}_{opt}$ [v_0]	S_{ropt} [%]	reality v_r [v_0]
two cars	1.684	3.732	2.112	0.698	80	2.171
swift car	1.630	2.474	1.751	0.685	100	1.813
slow car	0.952	1.780	1.067	0.392	40	1.032

Figure 7 shows the numerical results of the whole process. The first column contains the average velocity measured for each Ω, without any optimisation (in this case, $\omega = \Omega$ and $S_{ropt} = +\infty$). The second column shows the standard deviation associated to this computation. One can clearly see that the measurements are very noisy. The third and forth columns show the same results, but with application of the computed S_{ropt} threshold, which appears in the fifth column. One can see that the results have been greatly improved. The last column gives the reference speed v_r, measured by some other method. The concordance is satisfying. All units are referenced to an arbitrary speed v_0, choosen in such a way that:

(14) $$v_0 = \frac{\Delta X}{\Delta T}$$

Object	Ω L/2 [v_0]	n []	$\omega(S_{ropt})$ $L_{opt}/2$ [v_0]	n_{opt} []	Errors $\frac{\lvert\hat{v}_{opt}-v_r\rvert}{v_r}$ [%]	$\frac{L_{opt}}{2\cdot v_r}$ [%]
two cars	0.127	3276	0.0356	1535	2.71	1.64
swift car	0.103	2183	0.0360	1445	3.42	1.98
slow car	0.053	4162	0.0248	1004	3.39	2.40

Figure 8 shows the width $L(S_r, \gamma)$ of the confidence interval, where the confidence level γ has been arbitrarily choosen to be equal to 95%. The number of points taking part to the computations is also given. The relative error of measurement $|\hat{v}-v_r|/v_r$ can be compared to the relative uncertainty given by L_{opt}/v_r. One can see that the errors are quiet low, typically inferior to 5%. The predicted range of tolerance (at 95% confidence level) is somewhat too narrow to contain the measurements made, but the real speed v_r has also its own tolerance, which can be considered as of the same order of magnitude than the one provided by optical flow computations. Indeed, the two intervals do overlap.

5. CONCLUSION

We have developed a method for measuring velocity with a set of two line image cameras, assuming an unidirectional motion. This method yields an optimal average speed value for a Gaussian speed distribution, based upon a criterion of minimal confidence interval within a set of points. In practical experiments, the Gaussian hypothesis is confirmed, whilst the criterion has proven to be usefull and gives good results.

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