and replaced by the $4f_{sc}$ clock signal. For the $4f_{sc}$ sampling rate, the sampled $\cos 2\pi f_{sc} t$ and $\sin 2\pi f_{sc} t$ signals become $\cos(n\pi/2)$ and $\sin(n\pi/2)$ in digital domain. As the integer n increases, three values 1.0, 0.0, and -1.0 alternate between two sequences.

We can use these sampled signals as control clock for detecting two color components, R-Y and B-Y, to avoid the multiplication process. When the control logic $c(n)$ for the scheme as shown in Fig. 1 is "1", the input chrominance signal from the digital bandpass filter is passed and for the logic "0", the input is stopped. In the other case, the input is passed and complemented for the arithmetical operation. This multiplerless digital color detection scheme can be implemented as shown in Fig. 1.

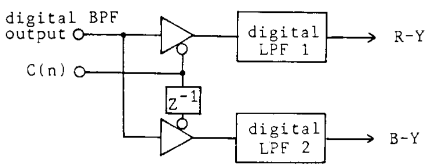


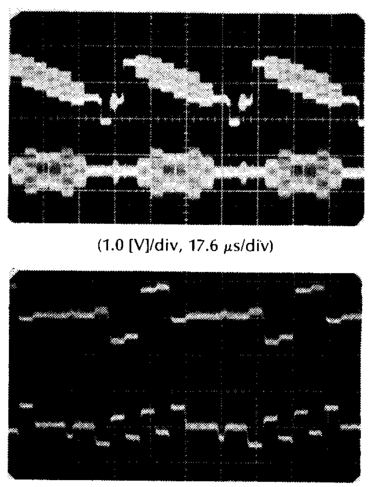
Fig. 1. Proposed digital color detector.

When the control clock is phase-locked with the color burst, R-Y signal lagged by 90° can be detected and B-Y signal also detected by one sample delay z^{-1}.

III. Results

A digital chrominance signal is obtained from the output of the digital bandpass filter (BPF) whose input is an A/D-converted NTSC video signal sampled at $4f_{sc}$ rate. The composite video signal (top) and the band pass filtered chrominance signal (bottom) are shown in Fig. 2(a).

Detected color components R-Y (top) and B-Y (bottom) are shown in Fig. 2(b).



(a) (1.0 [V]/div, 17.6 μs/div)

(b) (0.5 [V]/div, 14.8 μs/div)

Fig. 2. (a) Input/Output signals of BPF. (b) Detected color components.

IV Conclusion

The quadrature-multiplexed color components R-Y and B-Y modulated by the subcarrier signal have been easily demodulated by the phase-locked $4f_{sc}$ clock signal without resorting to digital multipliers. We have confirmed that the detection process can be realized in real-time via the hardware implementation.

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Comments on "A New Approach to Recursive Fourier Transform"

MICHAEL UNSER

The above letter[1] calls for two comments; the first relating to the newness of some of the results and the second relating to the practicality of the proposed algorithms. First of all, the recursive structure of the running Fourier transform has been investigated by a number of authors [1]-[6], none of whom is quoted by Amin. Furthermore, the main idea behind the generalization presented in the third section stems from the properties of a yet more general class of features that can be computed using the same recursive structure. This result is expressed by the following theorem to be found in [6].

Theorem: A feature $g(n)$, being a function of the sample values $x_n, \cdots, x_{n+N-1}$, satisfies the first order recursion condition

$$g(n) = w \cdot g(n-1) + w_+ \cdot G(x_{n+N}) + w_- \cdot G(x_{n-1}) \qquad (1)$$

where w, w_+ and w_- are complex values and $G(\cdot)$ is an arbitrary function, iff

$$g(n) = \sum_{m=0}^{N-1} w^{-m} \cdot G(x_{n+m}) \quad \text{and} \quad \begin{cases} w_+ = w^{-N+1} \\ w_- = -w. \end{cases} \qquad (2)$$

Examples of quantities sharing this property are the Fourier coefficient $F(n, \omega)$, the local mean value, the qth order moment, and the z transform (value $z = z_0$) (cf. [6], table 1). In particular, the exponentially weighted Fourier coefficient $F(n, \omega, \gamma)$ described by Amin is the z-transform evaluated at $z_0 = \gamma e^{-j\omega}$.

When discussing the issue of computational complexity, the author does not take into account the fact that the use of a windowing function produces running Fourier coefficients that are bandlimited and that there is no major loss of information when $F(n, \omega)$ is sub-sampled at a rate of $N/2$, which can result in a substantial saving in the number of operations when using a nonrecursive algorithm.

The author also considers the general form of a weighted running Fourier coefficient and suggests expressing the weighting function as a sum of $2M + 1$ geometric series and treating each term separately. There are two major drawbacks with this approach. First, there is generally no guarantee of the existence of such a decomposition. Second, as stated by the author, this method turns out to be quite impractical for large values of M.

When dealing with an arbitrary weighting function, there is an alternative and generally simpler approach which evaluates a given

Manuscript received January 20, 1988.

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IEEE Log Number 8823840.

[1] M. G. Amin, *Proc. IEEE*, vol. 75, no. 11, pp. 1537-1538, Nov. 1987.

Fourier coefficient by using a modulator followed by a low-pass filter. This design is based on the following decomposition:

$$F_w(n, \omega) = \sum_{m=0}^{N-1} w_m x_{n-m} e^{-j\omega m} = e^{-j\omega n} \sum_{m=0}^{N-1} w_m y_{n-m} = e^{-j\omega n}(w_n^* y_n) \tag{3}$$

where $\{y_n\}$ is the modulated signal:

$$y_n = x_n e^{j\omega n}. \tag{4}$$

In this approach, apart from a pre- and post-multiplication by a complex exponential, the computational complexity depends entirely on the efficiency of the convolution with the windowing kernel and is therefore directly related to the problem of efficient filter design. Usually, there is a fast algorithm (independent of the block size) whenever the windowing filter can be implemented recursively. For example, polynomial windows of degree $(q - 1)$ can be implemented from a cascade of q moving average filters which require no more than 2 operations per sample. This approach is not new and has been used for many years in analog spectrum analyzers.

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Author's Reply[2]

Many papers including those mentioned by Unser have been published in the area of recursive Fourier transform (RFT) and its implementation. These papers indeed share the same scope, yet their focus is different than that of [1] which briefly addressed the computationally block invariance (CBI) property of RFT, its generalization, and extention to the computationally lag-invariant recursive spectrum estimation problems. For this reason, as well as space limitation, we referenced in [1] two books for further readings on RFT.

The commonly used approach to RFT as a modulated signal convolved with a linear time-invariant filter is a useful tool in signal analysis and may be used for a different interpretation to the CBI-RFT property. The latter becomes a result of expressing the FIR filter in terms of a finite weighted sum of the IIR filters. When two IIR filters are used to design a finite duration sequence, which correspond to the theorem given in [2], the two filters must represent a pair of a single pole IIR filter and its delayed version, as it is the case with exponential or equal coefficient FIR. In this context, generalization [1, eq. 9] is equivalent to using multiple pairs of single pole filters. Examples which fall under this generalization are Hanning and Hamming windows.

Our letter did not address the issue of possible subsampling under the assumption of narrow low-pass characteristics of the employed window, since the task was not to compare the computations in recursive and nonrecursive calculations of the discrete Fourier transform. The results by Unser [2] on this subject, however, can be valuable in determining the proper approach to on-line Fourier-based spectral analysis.

In replying to the comment made about the impracticality of the generalization of the CBI-RFT, we note that the decomposition of a finite duration window to M different windows is not generally recommended for use in all applications. Equation (9) in [1] may first be used to qualify the window for CBI-RFT. If the window qualifies, its recursive or nonrecursive implementation must then be decided based on the value of the block length N in relation to M.

To reply to the last comment, we maintain that using $(N - 1)$ factors in the cascading structure realization of an N-samples FIR filter violates the CBI property, since it establishes a dependence between the total computations and the number of filter singularities.

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[2]Manuscript received April 22, 1988.

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Comments on "Closed-Form Solution for Underground Impedance Calculations"

JAMES R. WAIT

This letter presents solutions relevant to electromagnetic wave transmission along conductors in a cylindrical tunnel. The results and conclusions differ from previous work not referenced. In the above letter,[1] the authors formulate the boundary value problem of an electric line source eccentrically located in a cylindrical cavity. Their solution is consistent with the full-wave solution for a similar problem [1] arising in electrical geophysics. However, in reducing their solution to a simpler form, they make some physical approximations which are questionable.

Tylavsky, Brown, and Ma present a "new, more accurate approximation" for the modified Bessel function; it reads

$$K_0(z) = \ln(1 + 1/z). \tag{1}$$

For $|z| \ll 1$, it behaves as $-\ln z$ which, apart from a constant, does have the correct small argument form [2]. However, for $|z| \gg 1$, it behaves as $1/z$ while the correct limiting asymptotic form is $(\pi/2z)^{1/2} \exp(-z)$. The authors then assert that, for $\arg z = \pi/4$,

$$K_0(z)/K_1(z) \simeq z \ln(1 + 1/z) \tag{2}$$

is a valid representation for all values of $|z|$. Indeed they demonstrate that the percentage error, in using the approximation, is small for the condition on the phase angle of z. However, this agreement with the exact form is fortuitous because $K_1(z)$ is approximated, throughout the range of z, by $1/z$ which is only valid if $|z| \ll 1$. In fact, it we accept (1) then, to be consistent we must have

$$K_1(z) = -K_0'(z) \cong -\frac{d}{dz}\ln\left(1 + \frac{1}{z}\right) = +\frac{1}{z(1 + z)} \tag{3}$$

which behaves as $1/z^2$ as $|z| \to \infty$.

Conceding that (2) is a useful empirical representation, for $\arg z = \pi/4$ for all $|z|$, one could accept their approximation for the series impedance Z_s for the case where the axial conductor is centrally located in the tunnel. However, their approximated expression for the eccentric case is only valid for small values of the argument d/p of the modified Bessel functions of order n where $n = 1, 2, 3, \cdots$. Consequently their conclusion that the series imped-

Manuscript received December 28, 1987; revised February 18, 1988.

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IEEE Log Number 8823838.

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