Self-Similar Vector Fields

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SELF-SIMILAR VECTOR FIELDS

- Self-similarity:
  - What?
    - Geometric
    - Stochastic (Στοχασμός)
  - Why?
- What's a vector field?
- Our work:
  - Theory: Mathematical Models for self-similar flow
  - Practice: Reconstruction of flow from measurements

SELF-SIMILARITY

WHAT IS SELF-SIMILARITY?

WHAT IS SELF-SIMILARITY ?

SIMILARITY TO SELF :)

WHAT IS SELF-SIMILARITY ?

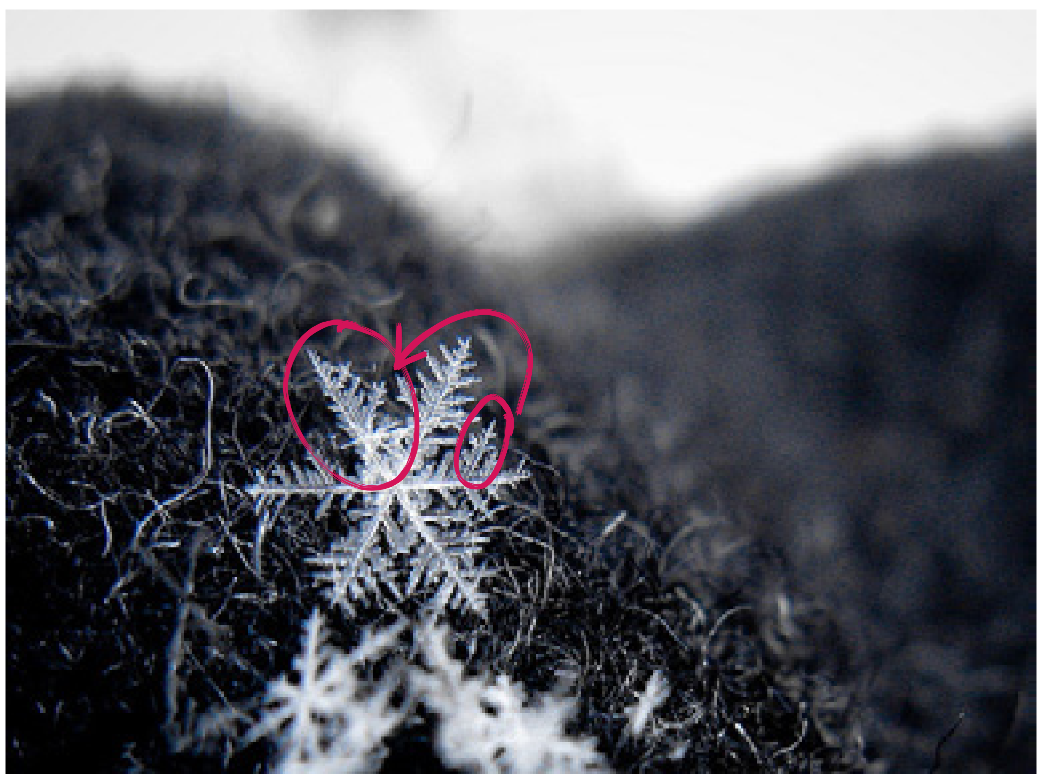
SIMILARITY TO SELF :)

«FRACTAL» SNOWFLAKE

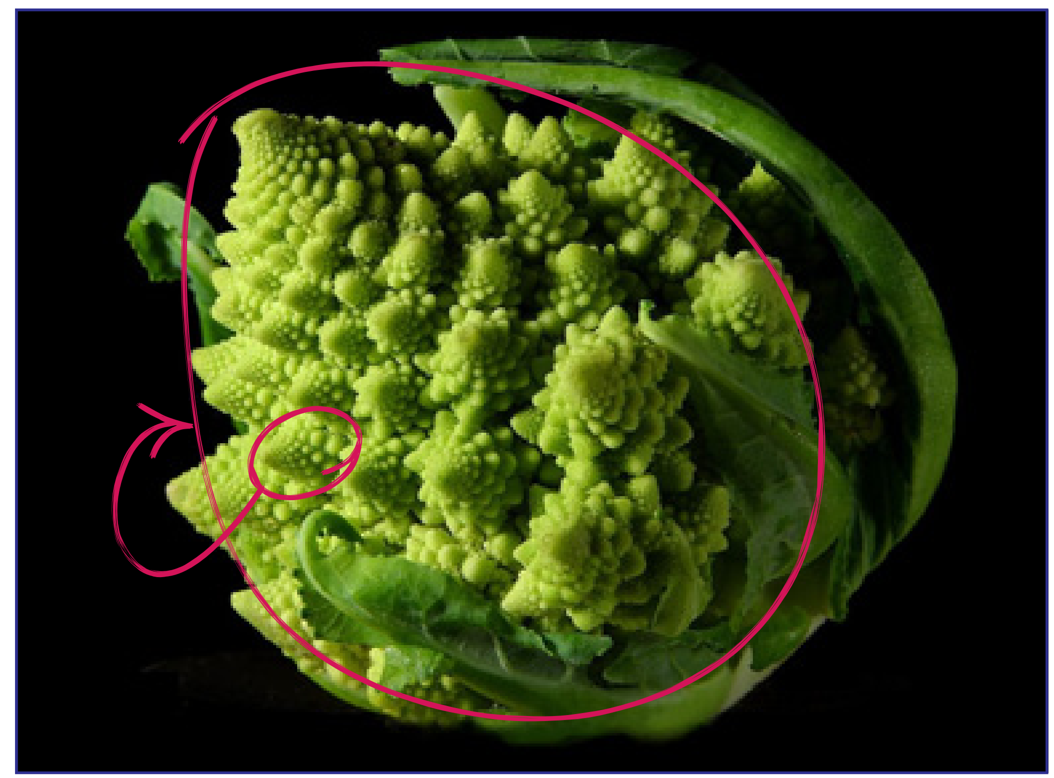
«FRACTAL» BROCOLLI

WHAT IS SELF-SIMILARITY ?

SIMILARITY TO SELF :)



«FRACTAL» SNOWFLAKE



«FRACTAL» BROCOLLI

WHAT IS SELF-SIMILARITY?

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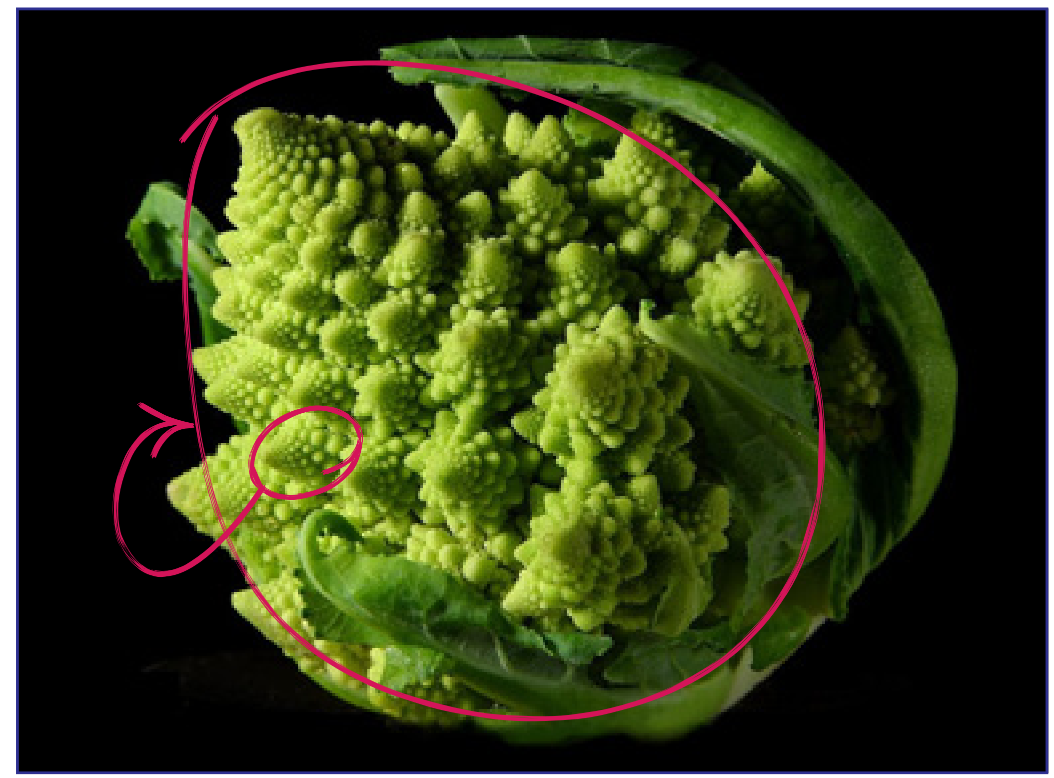


GEOMETRIC: SHAPE IS INVARIANT UNDER TRANSFORMATIONS

WHAT IS SELF-SIMILARITY?

X

THING

T

TRANSFORMATION

(ROTATION, RESIZING, etc.)

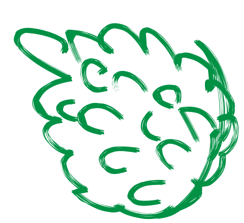
SAME THING

GEOMETRIC: SHAPE IS INVARIANT UNDER TRANSFORMATIONS

WHAT IS SELF-SIMILARITY?

$$X = TX$$

X THING $\xrightarrow[\text{(ROTATION, RESIZING, etc.)}]{T \text{ TRANSFORMATION}}$ SAME THING



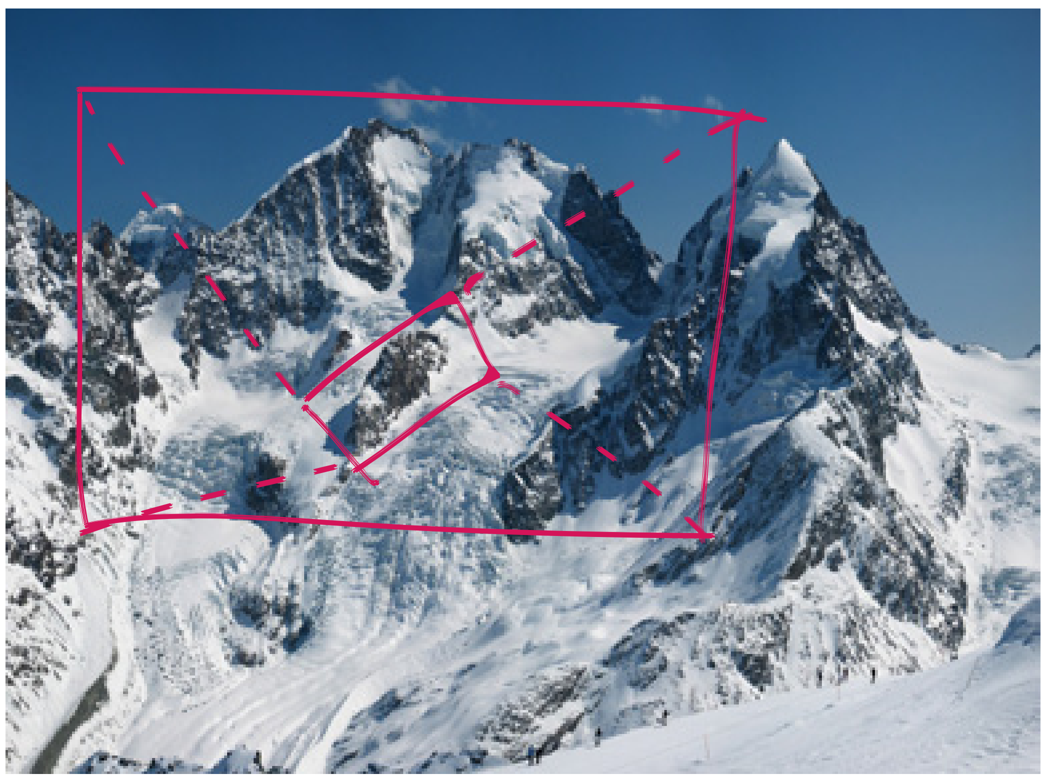
$\boxed{\text{GEOMETRIC:}}$ SHAPE IS INVARIANT UNDER TRANSFORMATIONS

WHAT IS SELF-SIMILARITY ?

ANOTHER KIND OF
SELF-SIMILARITY

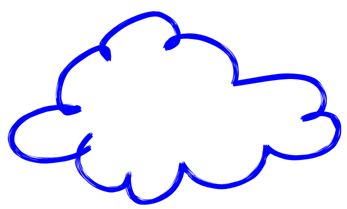
WHAT IS SELF-SIMILARITY ?

ANOTHER KIND OF
SELF-SIMILARITY



WHAT IS SELF-SIMILARITY?

THING

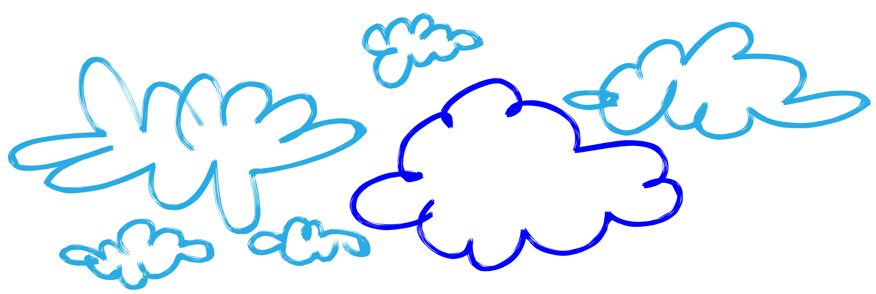


WHAT IS SELF-SIMILARITY?

THING

FROM an

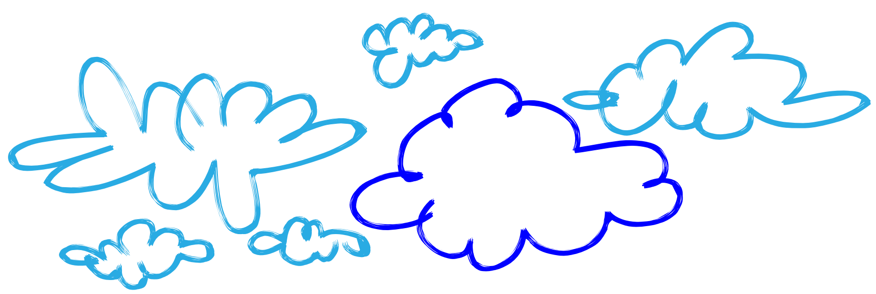
« ENSEMBLE OF THINGS »



WHAT IS SELF-SIMILARITY?

THING $\xrightarrow{\text{TRANSFORMATION}}$ ANOTHER THING

THING FROM an «ENSEMBLE OF THINGS»



ANOTHER THING FROM THE SAME ENSEMBLE

WHAT IS SELF-SIMILARITY ?

THING $\xrightarrow{\text{TRANSFORMATION}}$ ANOTHER THING

FROM an « ENSEMBLE OF MORE or LESS LIKELY THINGS »

FROM THE SAME ENSEMBLE

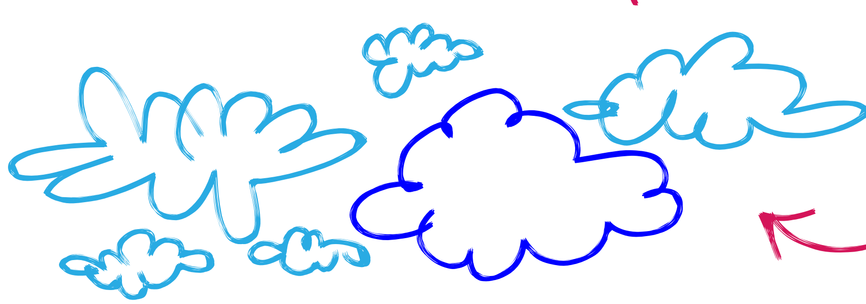
WHAT IS SELF-SIMILARITY ?

THING $\xrightarrow{\text{TRANSFORMATION}}$ ANOTHER THING

FROM THE SAME ENSEMBLE

FROM an MORE or LESS LIKELY

« ENSEMBLE OF THINGS »



PRETTY LIKELY

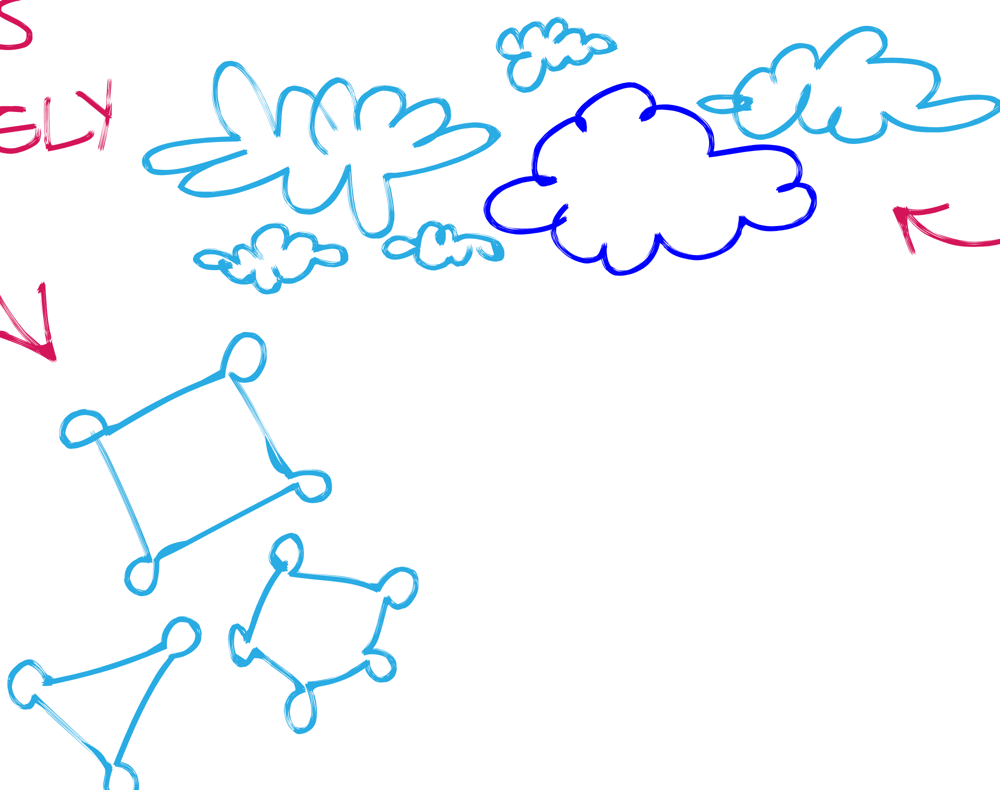
WHAT IS SELF-SIMILARITY?

THING —TRANSFORMATION→ ANOTHER THING FROM THE SAME ENSEMBLE

FROM an MORE or LESS LIKELY « ENSEMBLE OF THINGS »

LESS LIKELY

PRETTY LIKELY



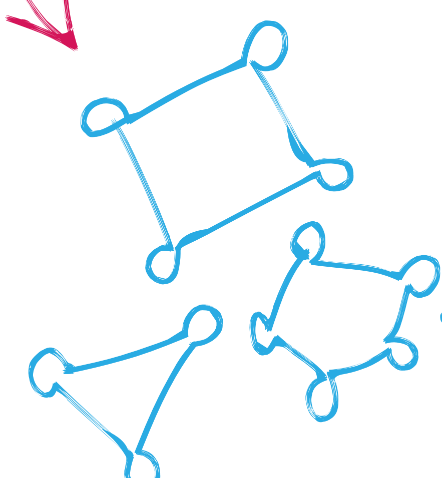
WHAT IS SELF-SIMILARITY?

THING $\xrightarrow{\text{TRANSFORMATION}}$ ANOTHER THING

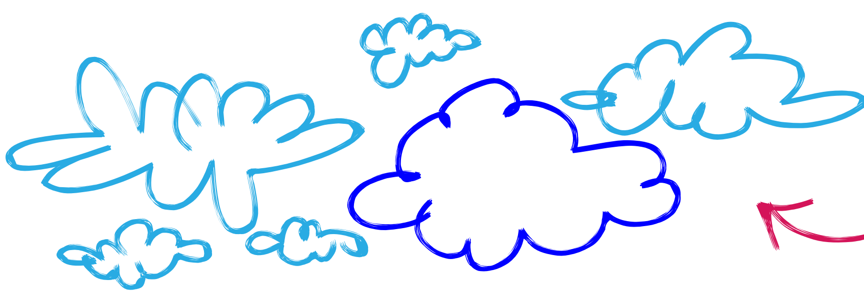
FROM AN «ENSEMBLE OF MORE or LESS LIKELY THINGS»

FROM THE SAME ENSEMBLE

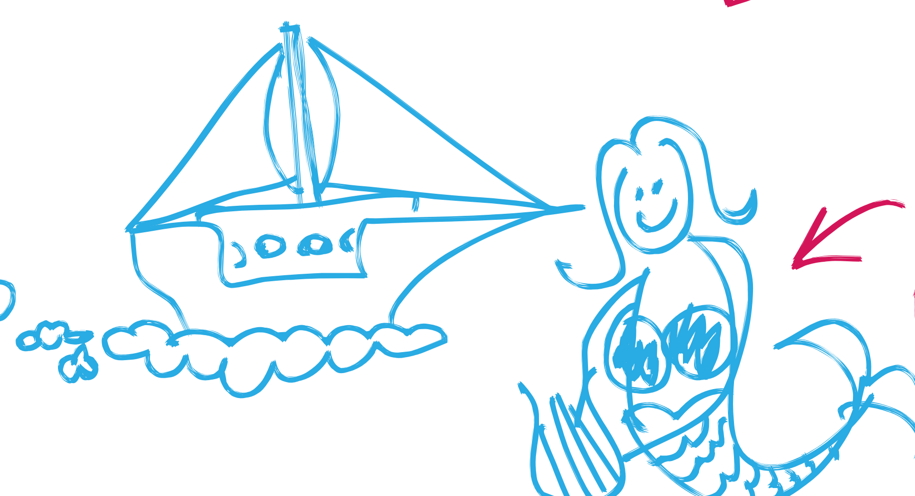
LESS LIKELY



PRETTY LIKELY



(SADLY) HIGHLY UNLIKELY



« PROBABILITY »

WHAT IS SELF-SIMILARITY ?

TRANSFORMATION

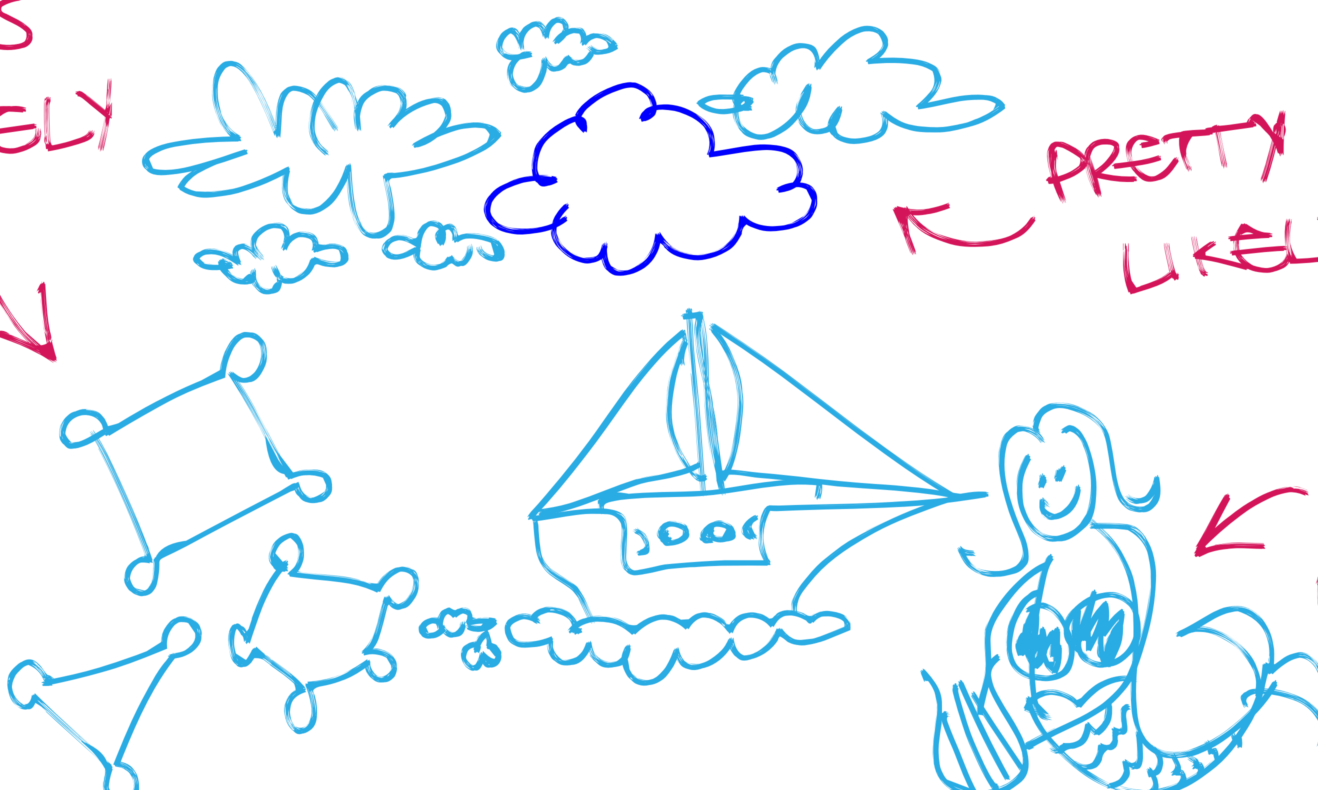
THING → ANOTHER SIMILARLY LIKELY or UNLIKELY THING FROM THE SAME ENSEMBLE

FROM an « ENSEMBLE OF MORE or LESS LIKELY THINGS »

LESS LIKELY

PRETTY LIKELY

(SADLY) HIGHLY UNLIKELY



« PROBABILITY »

LIKELY

WHAT IS SELF-SIMILARITY ?

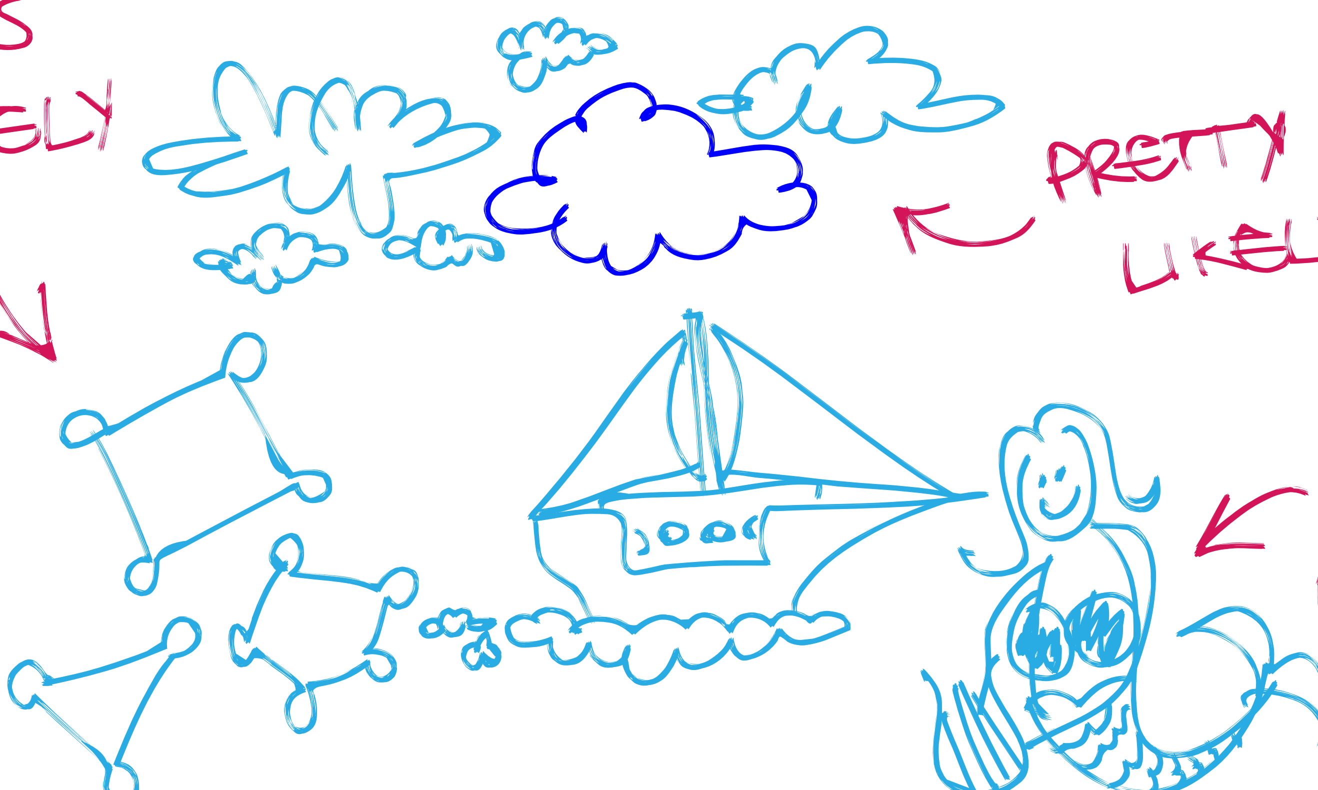
THING $\xrightarrow{\text{TRANSFORMATION}}$ ANOTHER THING

FROM AN MORE OR LESS LIKELY THINGS «ENSEMBLE OF THINGS»

SIMILARLY LIKELY OR UNLIKELY ANOTHER THING FROM THE SAME ENSEMBLE

LESS LIKELY

PRETTY LIKELY



(SADLY) HIGHLY UNLIKELY

« PROBABILITY »

LIKELY

NOT SO MUCH

WHAT IS SELF-SIMILARITY?

$$X = TX$$

WHAT IS SELF-SIMILARITY ?

$$X = T X$$ in probability law

$\boxed{\text{STOCHASTIC:}}$ ENSEMBLE PROBABILITIES ARE INVARIANT UNDER TRANSFORMATIONS

WHAT IS SELF-SIMILARITY?

$$X = TX$$ in probability law

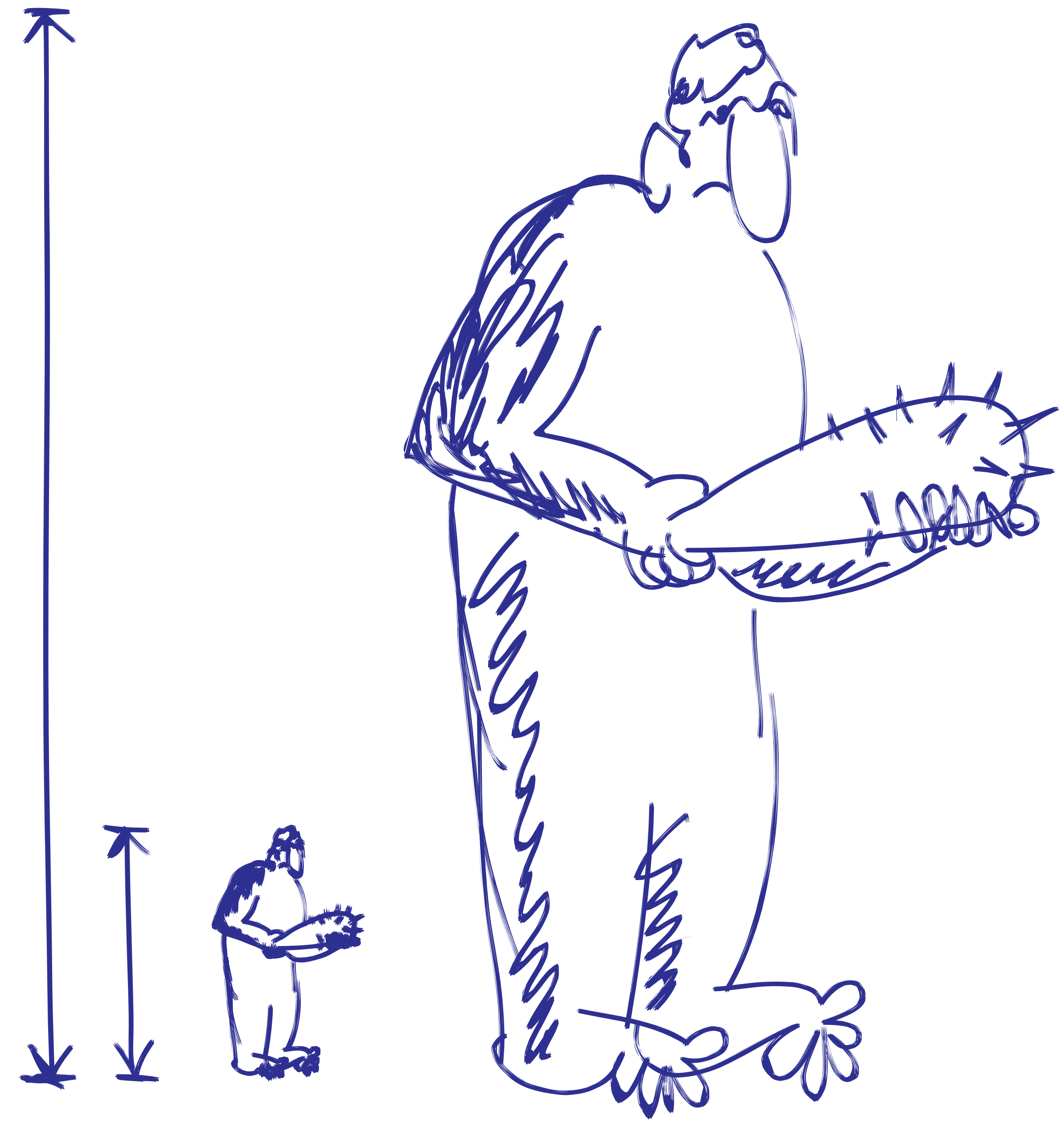
Στοχασμός

$\boxed{\text{STOCHASTIC:}}$ ENSEMBLE PROBABILITIES ARE INVARIANT UNDER TRANSFORMATIONS

WHAT TRANSFORMATIONS ?

WHAT TRANSFORMATIONS ?

-ROTATION-



-SCALING-

WHY SELF-SIMILARITY / INVARIANCE ?

WHY SELF-SIMILARITY/INVARIANCE ?

(PSEUDO-) PRINCIPLE of INSUFFICIENT REASON:

« If you don't have a reason to prefer one answer to another, give equal consideration to both »

WHY SELF-SIMILARITY / INVARIANCE ?

(PSEUDO-) PRINCIPLE of INSUFFICIENT REASON:

« If you don't have a reason to prefer one answer to another, give equal consideration to both »

all $\left\{\begin{array}{l}\text{directions}\\ \text{sizes}\end{array}\right.$ are equal

WHY SELF-SIMILARITY/INVARIANCE ?

(PSEUDO-) PRINCIPLE of INSUFFICIENT REASON :

« If you don't have a reason to prefer one answer to another, give equal consideration to both »

all { directions / sizes } are equal (although in practice some are more equal than others ...)

LISTEN TO SELF-SIMILAR "MUSIC"

http://bigwww.epfl.ch/tafti/gal/

(GO TO THE BOTTOM OF THE PAGE)

BACK TO THE TITLE...

SELF-SIMILAR VECTOR FIELDS

✓ ?

ROTATION- INVARIANT
SCALE-

VECTOR FIELDS

WHAT'S a FIELD?

WHAT'S a FIELD?

A REGION IN
SPACE and/or TIME ...

+

VALUES ATTACHED
TO POINTS IN THIS REGION

WHAT'S a FIELD?

A REGION IN

SPACE and/or TIME ...

Examples: This room,

an ice-skating rink

+

VALUES ATTACHED

TO POINTS IN THIS REGION

Examples: Temperature

Wind speed

WHAT'S a FIELD?

A REGION IN
SPACE and/or TIME ...

Examples: This room,
an ice-skating rink

+

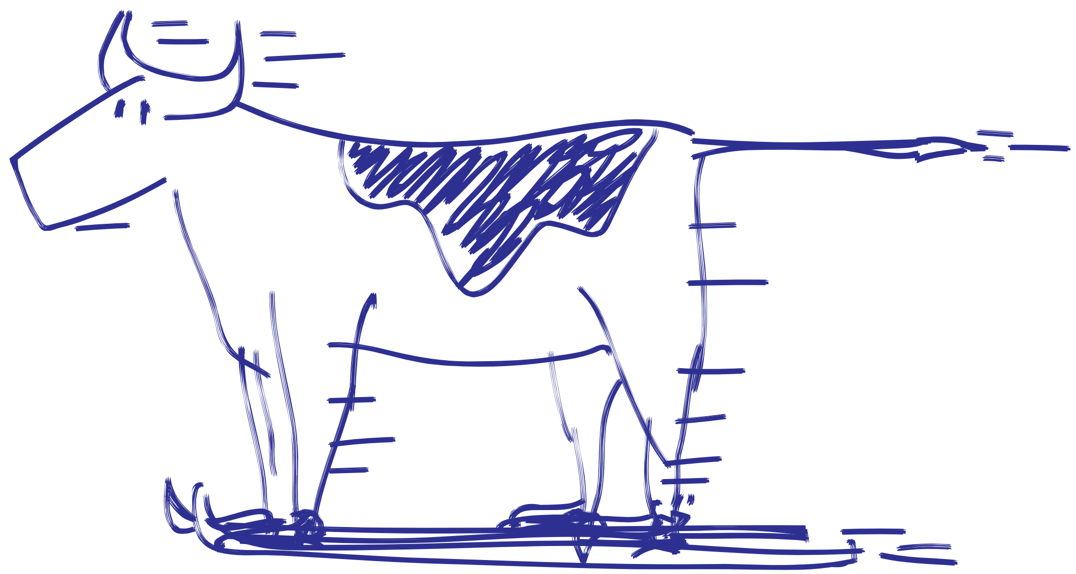
VALUES ATTACHED
TO POINTS IN THIS REGION

Examples: Temperature → SCALAR: No direction, just values
Wind speed → VECTOR: Values and directions

ROTATION OF VECTOR FIELDS

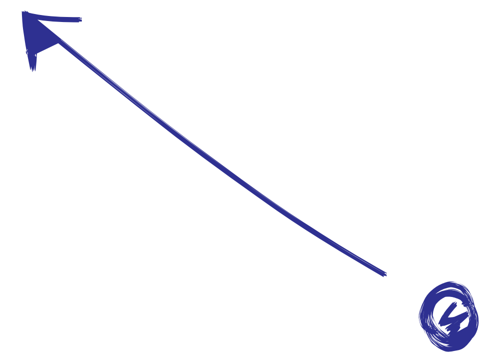
ROTATION OF VECTOR FIELDS

ROTATION OF VECTOR FIELDS



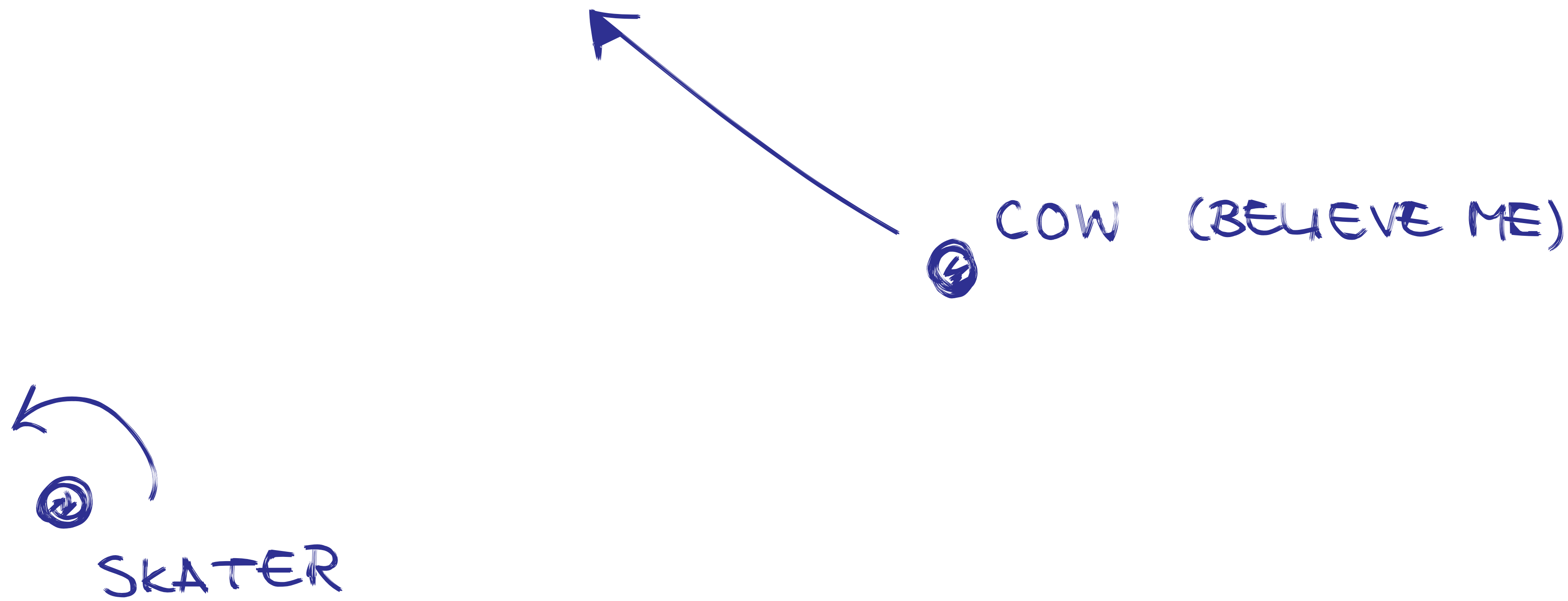
ROTATION OF VECTOR FIELDS

ROTATION OF VECTOR FIELDS



COW (BELIEVE ME)

ROTATION OF VECTOR FIELDS



ROTATION OF VECTOR FIELDS

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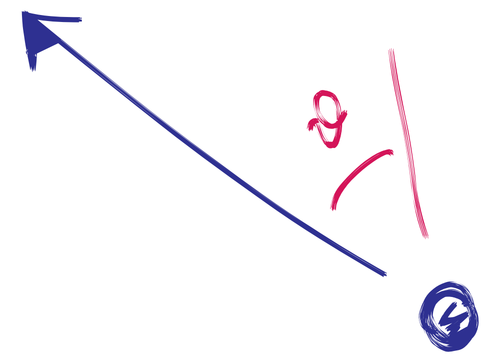

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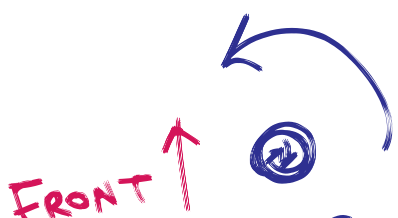
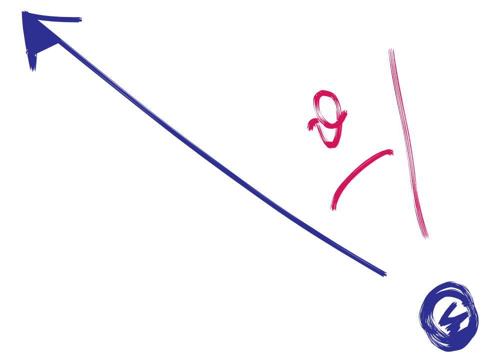


SKATER

$\text{speed} = (15\ \text{m/s}\ ,\ \vartheta)$

$\text{temperature} = -2.5\ °C$

ROTATION OF VECTOR FIELDS



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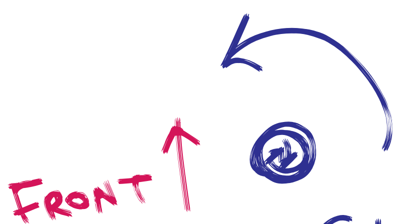


SKATER

speed $= (15\ m/s\ ,\ \vartheta)$

temperature $= -2.5\ °C$

VECTOR: Numerical representation <u>CHANGES</u> with rotation

SCALAR: " " <u>DOESN'T CHANGE</u> " "

OUR WORK

THEORY & PRACTICE

OUR WORK

1 THEORY

DEVELOP STOCHASTIC SELF-SIMILAR VECTOR FIELD MODELS

2 PRACTICE

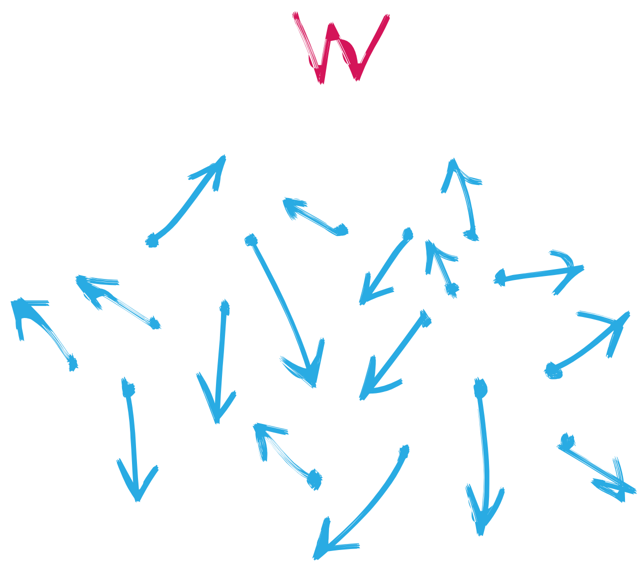
" SELF-SIMILAR ALGORITHMS TO RECONSTRUCT VECTOR FIELDS FROM DISTORTED MEASUREMENTS

THEORY

MATHEMATICAL SELF-SIMILAR MODELS

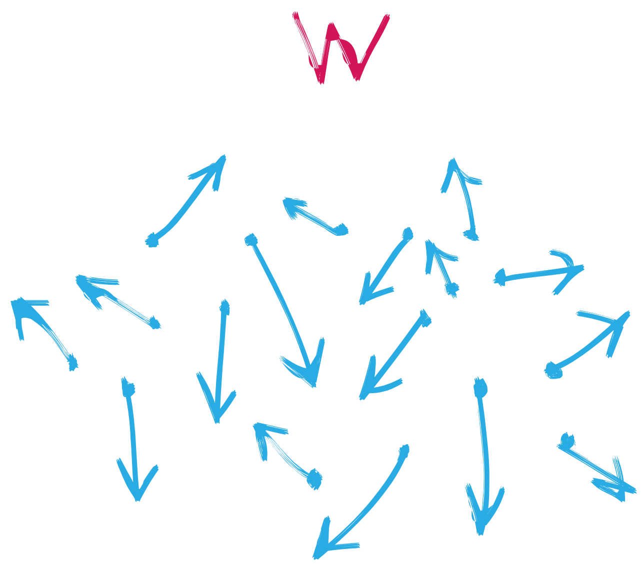
« In theory, there is no difference between theory and practice ...

MODELS



TAKE INDEPENDENT RANDOM VECTORS at EVERY POINT

MODELS

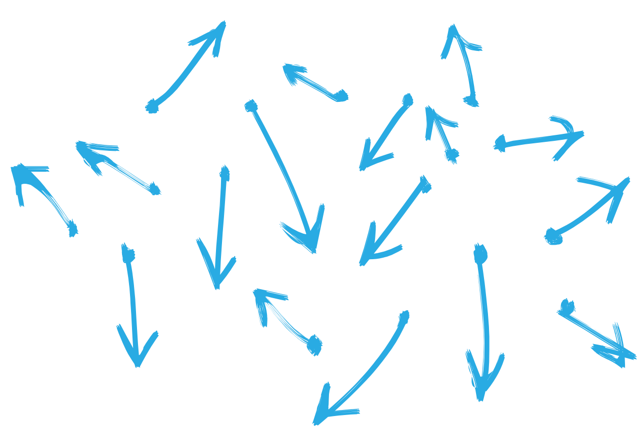


TAKE INDEPENDENT RANDOM VECTORS at EVERY POINT

MIX THEM USING a "SIMILARITY-PRESERVING" OPERATOR

MODELS

W



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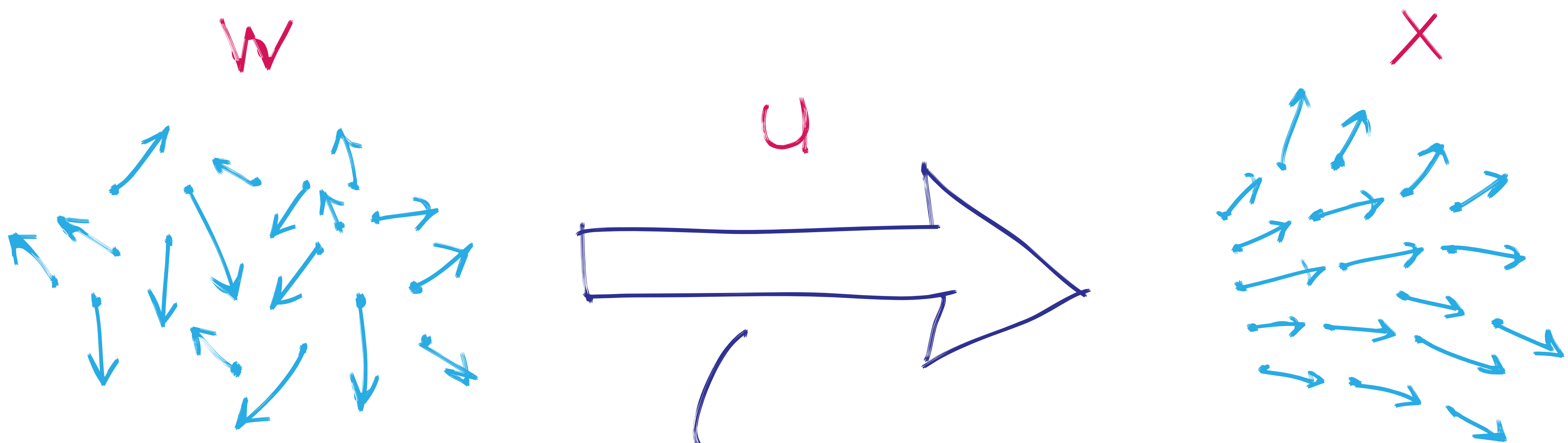
TAKE INDEPENDENT RANDOM VECTORS at EVERY POINT

MIX THEM USING a "SIMILARITY-PRESERVING" OPERATOR

- ADDS REGULARITY
- KEEPS $\left\{ \begin{array}{l} \text{ROTATION-} \\ \text{SCALE-} \end{array} \right.$ INVARIANCE

MODELS



TAKE INDEPENDENT RANDOM VECTORS at EVERY POINT

MIX THEM USING a "SIMILARITY-PRESERVING" OPERATOR

- ADDS REGULARITY
- KEEPS ROTATION- / SCALE- INVARIANCE

MODELS

THEOREM:

« We have them all »

MOREOVER, WE CAN CONTROL

ROTATIONAL BEHAVIOUR

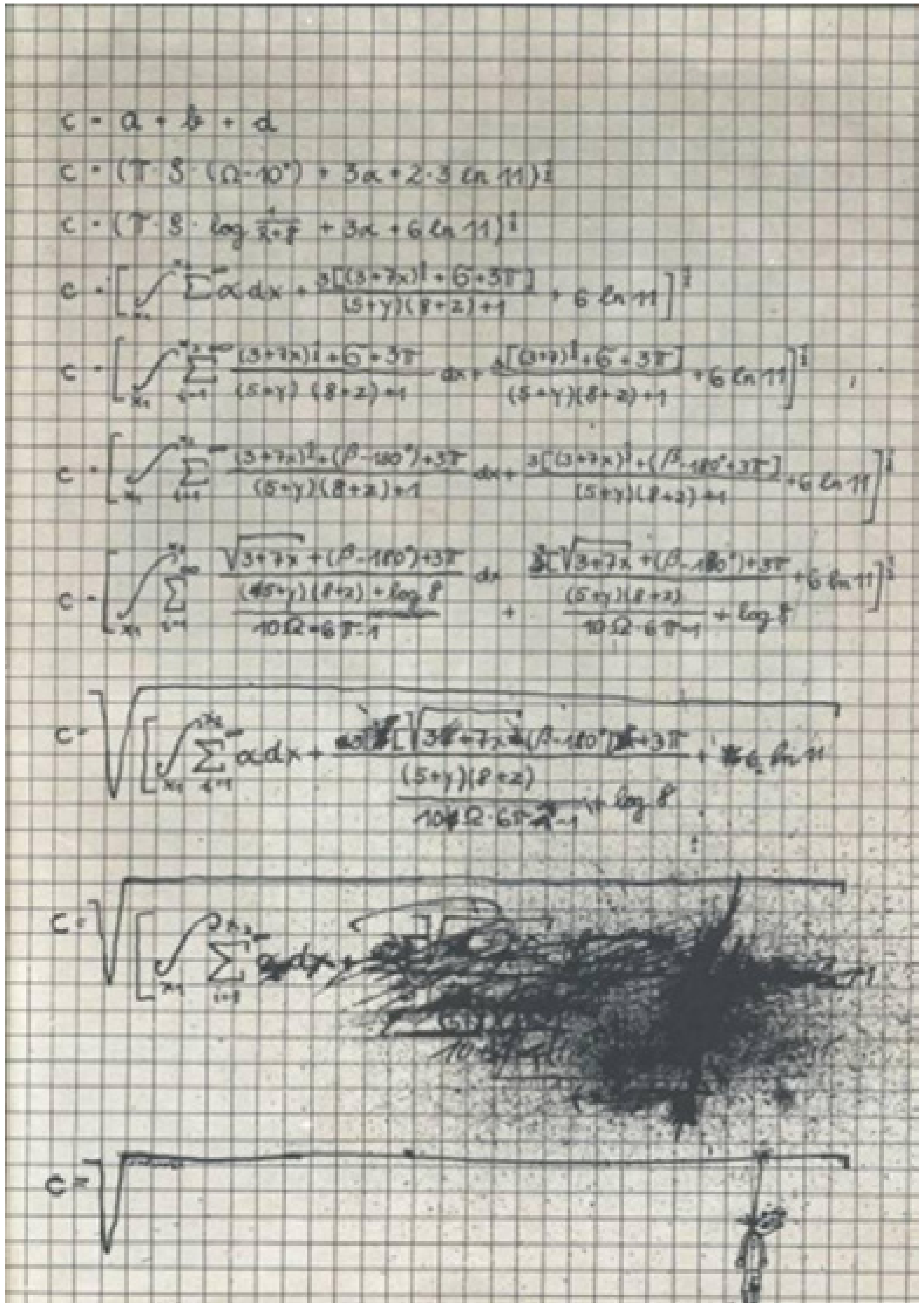
(WHIRLPOOLS)

<u>and</u>

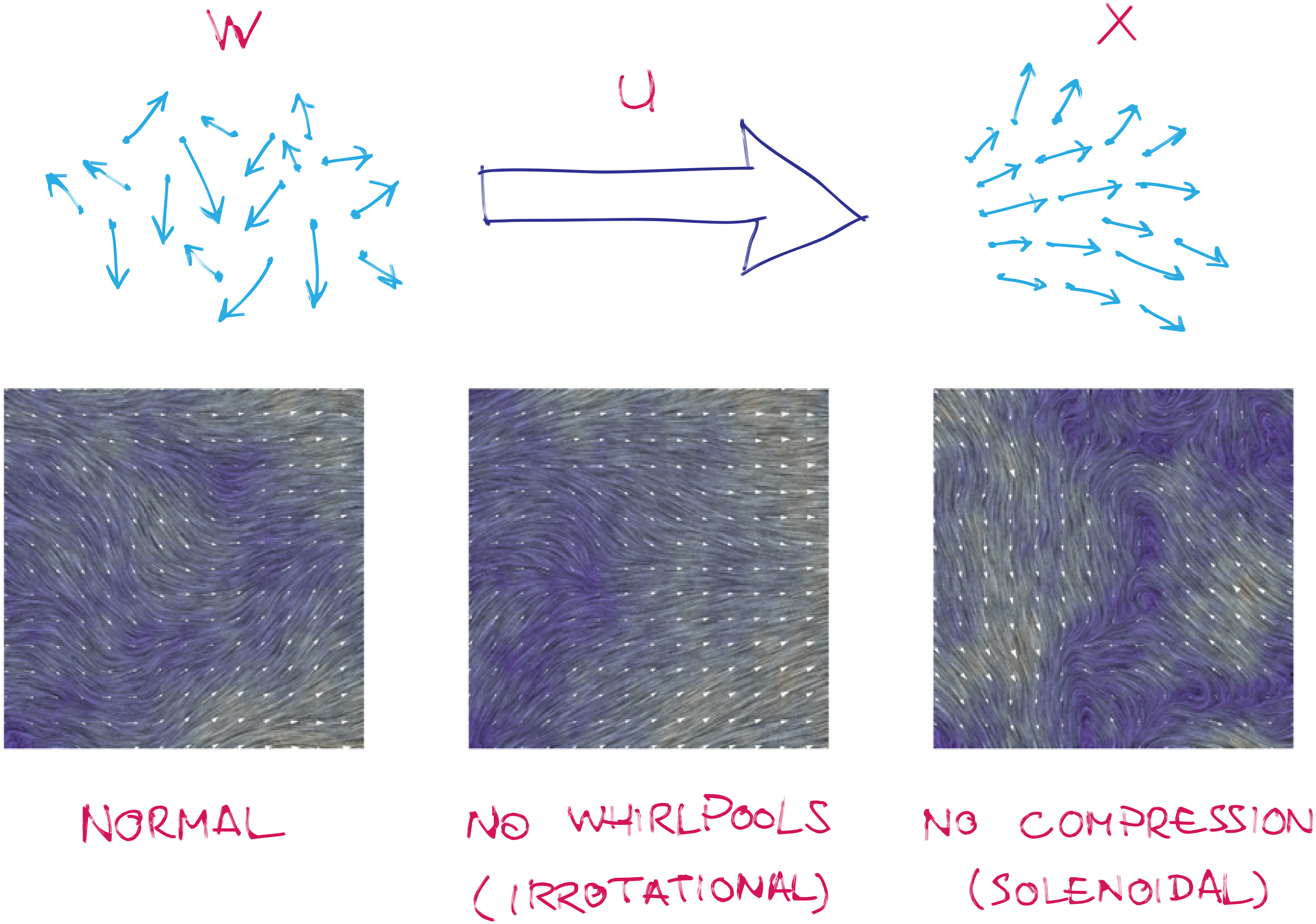
COMPRESSIVE BEHAVIOUR

(DENSITY)

PROOF :



Artist/author unknown


W
U
X
NORMAL
NO WHIRLPOOLS
(IRROTATIONAL)
NO COMPRESSION
(SOLENOIDAL)

PRACTICE

ENHANCING FLOW MEASUREMENTS WITHOUT ORIENTATION/SCALE PREFERENCE

« In theory, there is no difference between theory and practice ...

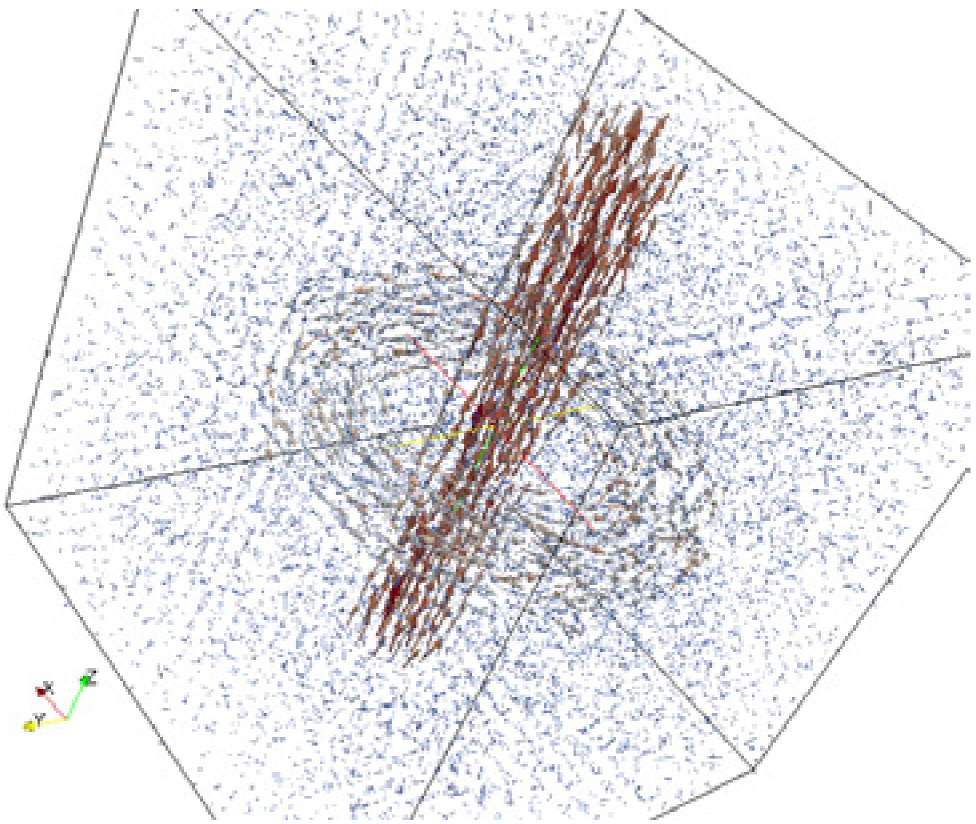
Practice

Enhancing flow measurements without orientation/scale preference

« In theory, there is no difference between theory and practice...

... but in practice, there is. »

RESULTS



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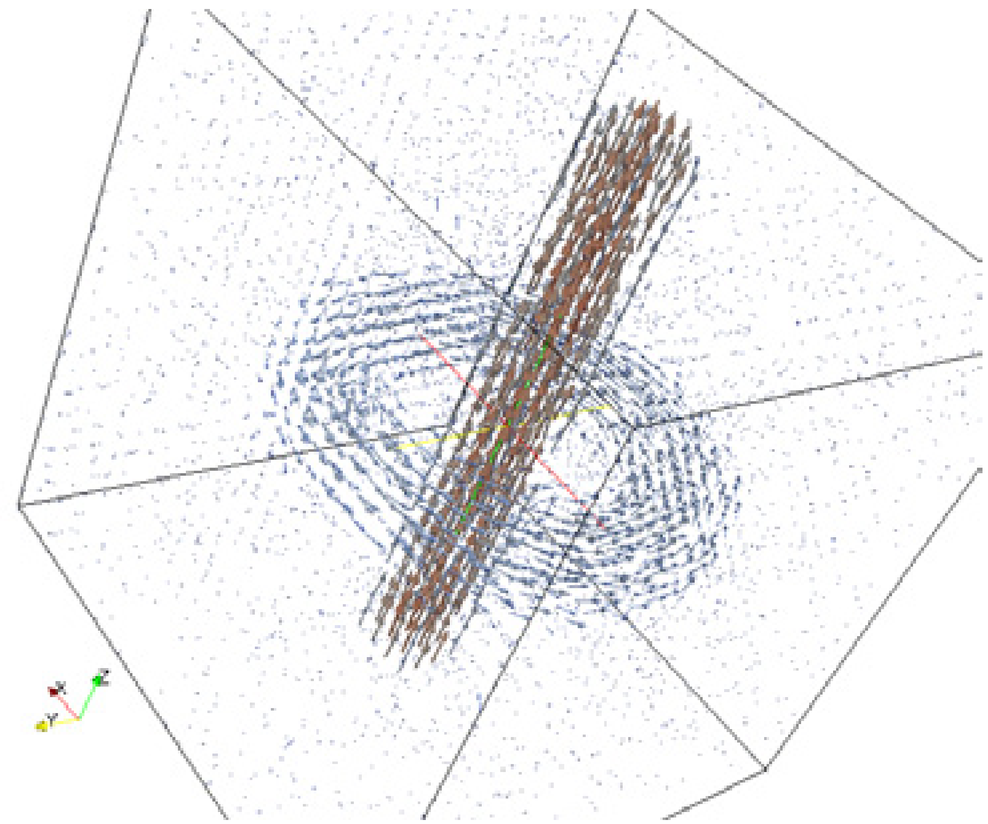
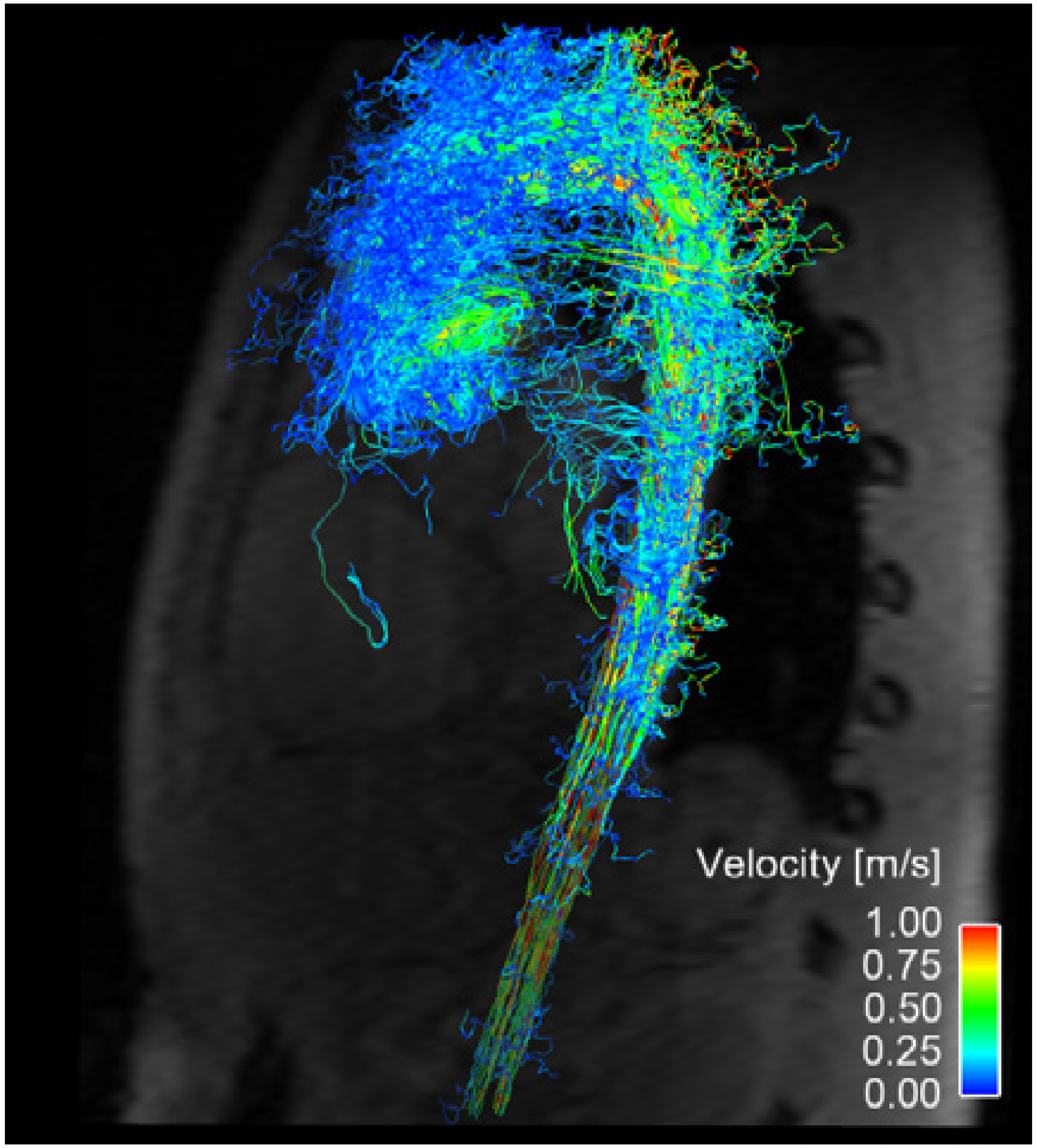


ENHANCED

RESULTS



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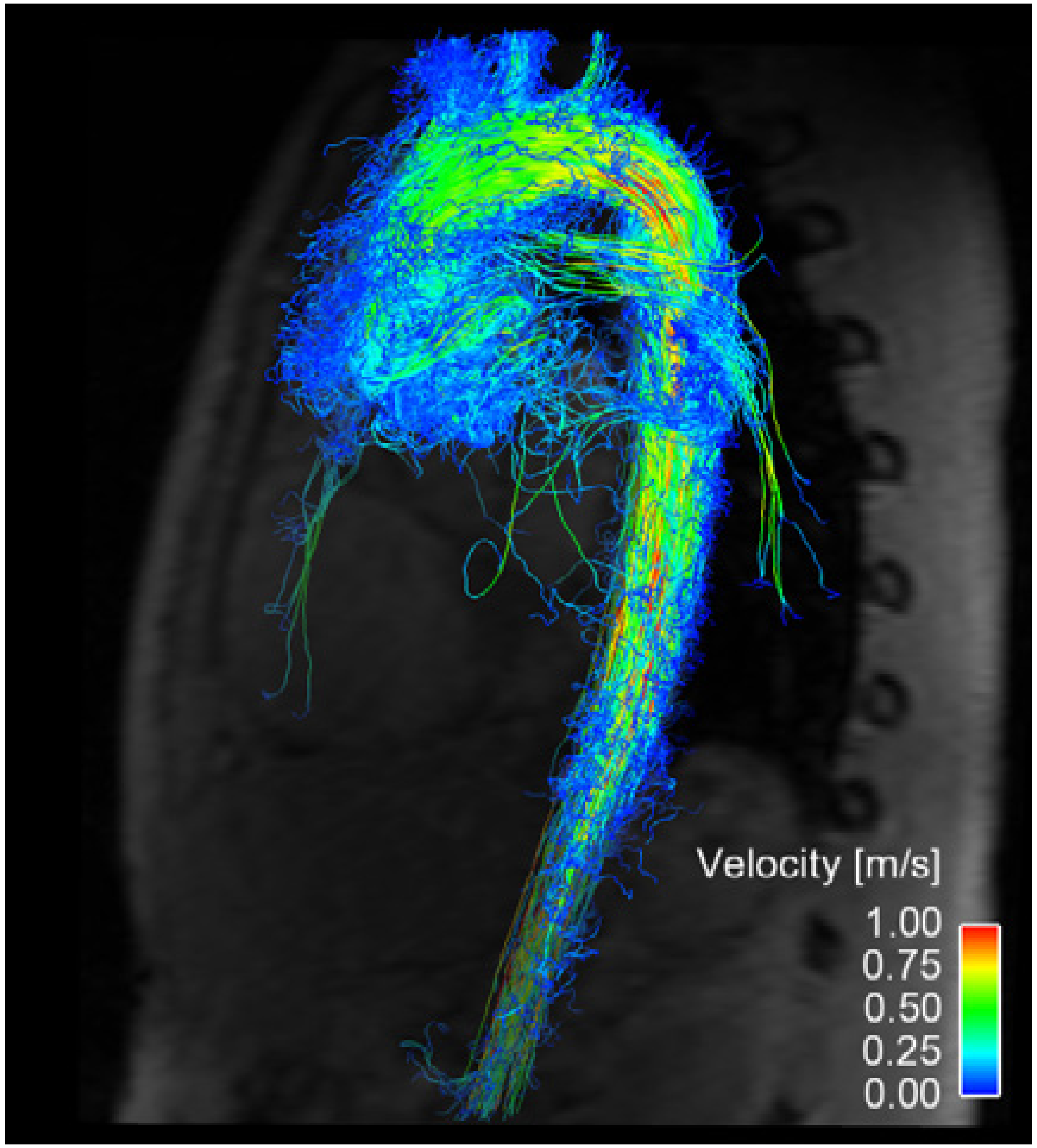

ENHANCED

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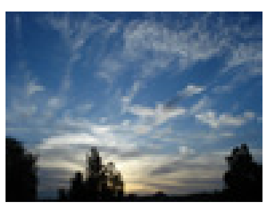
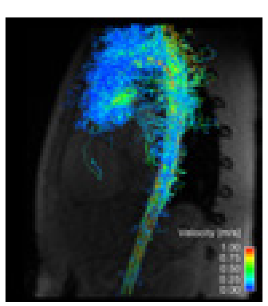
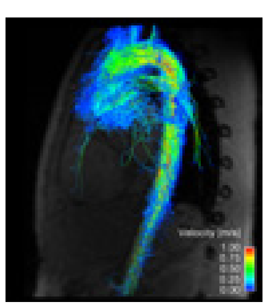

Chevy111, WP (GNU)


UNKNOWN



Dr A.F. Stalder (data & vis.)

THANK YOU!