

$$\sum_{k=0}^{K-1} x[k] e^{-j n \frac{2 \pi}{K} k} = \mathcal{F}_{k < K} \{x[k]\}[n] = \mathcal{F}\{x\}[n] = X[n].$$