Image resampling using least-squares spline models

Dimitri Van De Ville

Abstract— **From the very beginning of digital image processing, resampling has been a necessary proces to convert images to a different lattice. Simple algorithms using nearest-neighbor interpolation or bilinear interpolation are common practice and might suggest that it is an easy part of the image processing system. Unfortunately, the appearance of artifacts due to resampling, i.e., moiré patterns in printing, are often noticed and prove the necessity of providing sufficient attention to the image resampling stage.**

A good resampling algorithm needs to take into account the properties of the original and the new lattice. In this paper, we present an extension of the classical spline models to two-dimensional non-separable lattices. Next, we use this new model, which can be used to represent images on both rectangular and hexagonal lattices, to derive a least-squares interpolation function. The result is a convolution-based resampling algorithm. Experimental results for a practical printing application show that the interpolation function combines edge-preservation and moiré-suppression.

Keywords— **Image resampling, Moiré patterns, Edge-preservation, Convolution-based resampling**

I. Introduction

PROBABLY the best known theorem in digital signal processing is the Whittaker-Shannon sampling theorem [1]. This theorem states that in order to be able to reconstruct a continuous function out of uniform samples, the sampling rate must be at least twice the highest frequency present in the original signal. If this requirement is not satisfied, a phenomenon frequently referred to as *aliasing*, prevents the signal from being (completely) reconstructed. The range of allowable frequencies are contained into the so-called Nyquist range.

Although this theorem suggest that one must take into account the Nyquist range of the new lattice when resampling, typical and widespread resampling techniques for image processing, such as nearest-neighbor interpolation and bilinear interpolation, do not take into account the target lattice. Therefore, artifacts due to aliasing, such as undesirable moiré patterns (especially in printing) might arise. Further on, the advent of high-quality scanners and digital photography, with their high resolutions, increases the possibility that these artifacts appear.

The reasons of the popular usage of these interpolation functions are for one thing their ease of implementation, but for another the difficulties associated with the solution suggested by the sampling theorem itself. Indeed, the "optimal" interpolation function, corresponding to the ideal filter of the Nyquist range, has an unlimited support. Its sharp cut-off in the frequency domain brings about ringing artifacts, and an approximation on a limited support is difficult due to the slow decay of the sinc-function.

Dimitri Van De Ville is with the Department of Electronics and Information Systems (Elis), Ghent University, Gent, Belgium. E-mail: Dimitri.VanDeVille@rug.ac.be. He is a Research Assistant of the Fund for Scientific Research - Flanders (FWO, Belgium).

An elegant solution was proposed by Unser et al. [2, 3] for one-dimensional signals. Using two spline signal models, one suitable for the original lattice, another for the new lattice, they derived the interpolation function which realizes the minimal error between both representations in a least-squares sense. In this paper, we present an extension of the spline model for two-dimensional non-separable lattices (which cannot be treated by the tensor-product extension of Unser et al.). Next, we derive the least-squares solution according to these new models. Finally, we will show some experimental results for a printing application which demonstrate the feasibility of the proposed approach.

II. Generalized spline signal model

A continuous/discrete model allows us to construct a "smooth" signal based on the samples. Splines are a family of basis functions, which have a limited size of support, and expands as the order of the spline model increases. One of the most important spline families are the B-splines: piecewise polynomial functions which are symmetric. They are not orthogonal, but they form a Riesz basis and satisfy the partition of unity condition. It is also interesting to mention the convolution property, which enables us to construct splines of the next order by convolving the spline with the first-order spline. Note that first-order spline interpolation is better known as "nearest neighbor" interpolation; second-order spline interpolation as bilinear interpolation.

These models are appropriate for one-dimensional signals and can be extended to two-dimensional rectangular lattices by means of the tensor-product. We propose to construct a spline basis suitable for general periodic lattices. As an illustration, let's consider a regular hexagonal lattice. Since we are especially interested in preserving the convolution property (because it plays an important role in the derivation of the least-squares approximation) we apply it as a construction rule. As such, we first define the first-order hexagonal spline as the indicator function of the Voronoi cell of the lattice. For example, Fig. 1 (a) shows the first-order hexagonal spline. Note that it fills up the two-dimensional space if it is copied upon each lattice site (i.e., the partition of unity condition is fulfilled). A convolution of this spline with itself (and a proper normalization) results into the second-order spline, shown in Fig. 1 (b). We have proven that this spline family fulfills the necessary conditions to be a sensible continuous/discrete model. Additionally, the order of approximation corresponds to the nomenclature we introduced. An analytical expression was derived up to and including the third-order hexagonal spline.

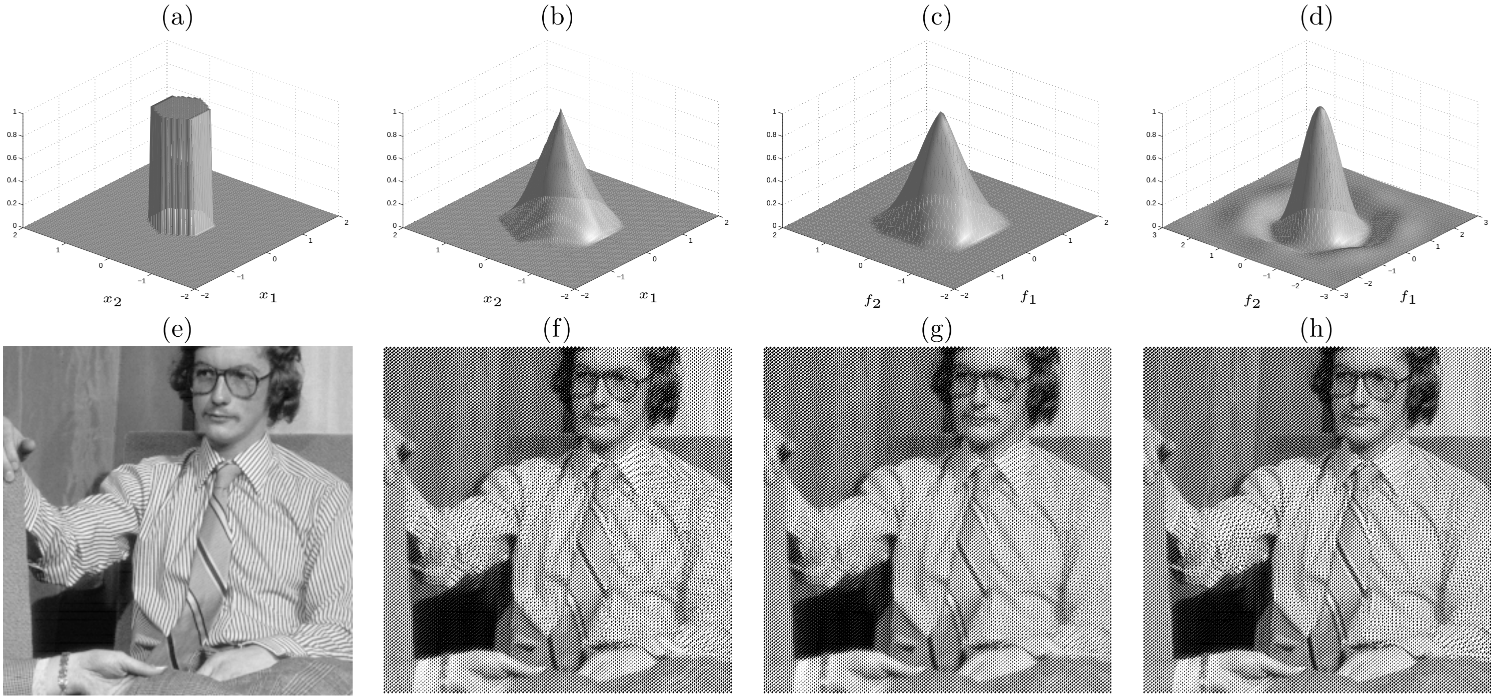

Fig. 1. (a) The first-order spline function for a hexagonal lattice. (b) The second-order spline function for a hexagonal lattice. (c) The least-squares interpolation function based on the first-order spline model for resampling from an orthogonal to a hexagonal lattice. (d) The second-order least-squares interpolation function. (e) Original test image. (f) Test image after halftoning using bilinear interpolation. (g) Test image after halftoning using first-order least-squares interpolation. (h) Test image after halftoning using second-order least-squares interpolation.

III. Least-squares resampling

The continuous/discrete model can now be used to reconstruct a continuous "surface" using the samples given on the lattice. Consider an original rectangular lattice (for which we can obtain a model using the same principle as explained before) and a new hexagonal lattice. The interpolation function we have derived minimizes the squared error between the representations implied by the signal models on both lattices. The order of both models can be chosen freely.

If we prefer first-order models, we obtain the interpolation function given in Fig. 1 (c). This most simple least-squares approach corresponds to "surface projection": the contribution of a sample value on the original lattice to a sample on a new lattice site corresponds to the relative overlap of their Voronoi cell's surface area. Note the difference with classical first-order interpolation (nearest-neighbor interpolation), which would simply assign the value of the sample of the nearest original lattice site.

From the second-order models, the least-squares interpolation functions do not have a theoretical unlimited size of support anymore. However, Fig. 1 (d) shows that the decay is much faster than sinc-like functions, making an approximation practical. We have also shown by the frequency analysis of the interpolation function, that these functions incorporate the Nyquist range of the new lattice. Current research investigates if it possible to implement this approach using recursive filters to get around the unlimited support problem.

IV. Some experimental results

Allmost all printing devices are bi-level, i.e., they are only able to produce black and white, they must use halftoning techniques to represent a continuous tone image (contone) by a bi-level image (halftone). When such a halftone is viewed by a human observer, the human visual system integrates the small bi-level features and creates the illusion of the original contone. The most popular halftoning technique is amplitude modulation: dots of varying sizes are placed on a periodic lattice. Resampling is required to obtain samples on this new lattice.

To demonstrate the feasibility of our least-squares resampling approach, we consider the image shown in Fig. 1 (e), which is resampled to the hexagonal lattice used by the halftoning proces. Figure 1 (f) shows the result after resampling by bilinear interpolation. Clearly, annoying moiré patterns appear in the shirt. The result after first-order least-squares resampling in (g) has already less moiré artifacts. Finally, the second-order least-squares resampling in (h) also preserves edges very well.

References

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