Sparse Image Deconvolution with Message Passing

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***Abstract*—We introduce an approximate message passing (AMP) algorithm for the problem of image deconvolution. The recovery problem is formulated in Bayesian terms, and uses sparse statistical priors for estimating the minimum-mean-squared-error solution. Our setting differs from previous investigations where AMP was considered for sparse signal recovery from random or Fourier measurements. AMP is incompatible with the original structure of the system matrices involved in deconvolution problems. We propose to recast the problem into a more suitable form, exploiting the fact that the convolution operator is diagonalized in the Fourier domain. This provides a remarkably effective deconvolution technique, which achieves significant improvement over ℓ_1-based methods, both in speed and quality.**

I. Overview

The basic deconvolution problem consists in the recovery of an image $\mathbf{x}$ from degraded linear measurements $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where the *system matrix* $\mathbf{H}$ models the response of the imaging system, and where $\mathbf{n}$ is an additive white Gaussian noise of variance σ^2. Problems of this nature arise in a wide variety of applications, including biomicroscopy, astronomy, and medical imaging.

In practice, the restoration problem is ill-posed and requires the introduction of additional prior information on the unknown $\mathbf{x}$ to obtain a meaningful solution. The literature on suitable priors for modeling natural signals is vast; however, one may single out ℓ_1-based and various Bayesian approaches. Here, we characterize $\mathbf{x}$ as one realization of a known random process that has sparse and independent components in some known transform domain.

Under the Bayesian model, the restoration process reduces to exact or approximate computations of standard statistical estimators. The estimator used most commonly is maximum a posteriori (MAP), which finds the most probable image that is consistent with the measurements. An alternative point of view is to develop an image-restoration method based on the computation of the minimum mean-squared error (MMSE) estimator. While MMSE estimators are generally intractable, they can be approximated with a class of algorithms called *approximate message passing* (AMP) [1].

Our contribution in this work is to demonstrate that a suitable adaptation of AMP can provide significantl performance improvement over traditional methods for the deconvolution of sparse images. The estimation performance of AMP is sensitive to the structure of the system matrix $\mathbf{H}$. It has been proven that AMP converges when $\mathbf{H}$ is a suitably rescaled random matrix [2], [3] and empirical observations also indicate favorable results for Fourier-type matrices [4], [5]. In our case, however, $\mathbf{H}$ corresponds to a space-invariant convolution, which makes the recovery ill-suited for AMP. Fortunately, the problem can be recast to an appropriate form for AMP reconstruction, exploiting the fact that the convolution operator is diagonalized in the Fourier domain.

II. Simulations

In our simulations, we consider the restoration of a sparse image of size 256×256 whose ratio of nonzero samples is equal to $\rho \in (0, 1)$.

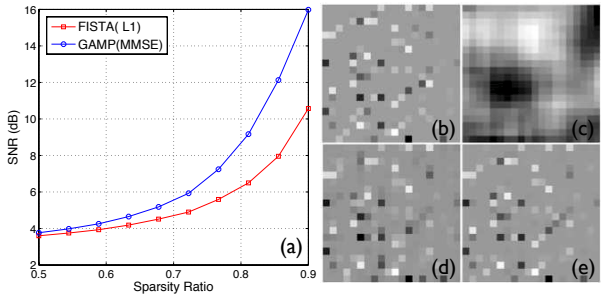

Fig. 1. Deconvolution of a sparse Gauss-Bernoulli image composed of 256×256 samples. (a) The average reconstruction SNR is plotted against the ratio of non-zero components of the signal. (b) Original, (c) convolved, (d) ℓ_1-regularization, (e) AMP.

Each nonzero value is drawn from the normal distribution of variance $1/(1-\rho)$. Our measurements consist in the convolution between the original image and a 9×9 Gaussian kernel of variance 3 with the addition of additive white Gaussian noise (AWGN) of variance $\sigma^2 = 10^{-5}$.

In Figure 1(a), we compare the deconvolution performance of AMP with the above Gauss-Bernoulli prior against the ℓ_1-regularized least-squares reconstruction that was implemented based on FISTA [6]. The regularization parameter λ of the ℓ_1 method was optimized for best MSE performance. We ran 10 iterations of AMP and 1000 iterations of FISTA, and plotted the average signal-to-noise ratio (SNR) of the reconstruction against the sparsity ratio ρ over 10 random trials in Figure 1. The results illustrate that AMP significantly outperforms ℓ_1-based FISTA over the whole range of sparsity ratios. A particular instance of the experiment for $\rho = 0.8$ is illustrated in Figure 1 (b)–(e), where the first 20×20 pixels from the top-left corner of the image are shown. This visual result corroborates the SNR values; the reconstruction obtained with AMP is sharper than the one obtained with the ℓ_1-based approach. Note that the cost per iteration is roughly the same for AMP and FISTA, the dominant elements being the application of $\mathbf{H}$, $\mathbf{H}^T$, and the componentwise shrinkage. However, AMP requires substantially fewer iterations, which allows for convergence in less than a second.

References

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