

for systems of linear equations, to name a few. They have many applications, in particular, in physics and chemistry.

The aim of this book is to give an overview of these questions and to show that a profound understanding of asymptotic expansions is necessary to comprehend extrapolation methods seriously. The author has himself contributed significantly to both subjects.

The first part of the book deals with asymptotic expansions: asymptotic systems and expansions, geometric asymptotic expansions, and logarithmic asymptotic expansions. The second part of the book is devoted to linear extrapolation methods: fundamental concepts and general philosophy, error bounds, stopping rules and monotonicity, generalizations, and final remarks. Historical notes and numerical examples have also been included.

This book is well written, well presented, easy to read, and, which is not so common, well printed. It is an important addition to the literature (which is not so abundant) on these topics. It could be used for a course on these questions and it is highly recommended to researchers and to all those who need to use convergence acceleration methods.

Claude Brezinski

E-mail: Claude.Brezinski@univ-lille1.fr

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Akram Aldroubi and Michael Unser, Eds., *Wavelets in Medicine and Biology*, CRC Press, Boca Raton, FL, 1996, 616 pp.

Wavelets have built a strong reputation in the context of signal and image processing. The editors of this book have invited several specialists to contribute a chapter illustrating this in the (bio)medical and biological sciences.

The book contains four parts. Part I has two chapters written by the editors themselves giving what they call a "surfing guide" to the theory and implementation of the wavelet transform. These 70 pages are among the better introductions to wavelets available in the literature. Both the continuous and the discrete wavelet transform are dealt with, but in view of the applications to follow, the emphasis is on the latter.

The second part deals with medical imaging and tomography. Mathematically speaking, tomography refers to the reconstruction of an image from (noisy) observations or approximations of line integrals which are often called projections. This problem is well known to be ill posed and thus very sensitive to noise. Therefore, a constant observation made in these chapters is that by taking the wavelet transform, it becomes easier to distinguish noise from the clean data. Noise is typically small, of high frequency, and uncorrelated, while the locality of the wavelet basis in both the space and the frequency domains allows one to catch the true image in only a "few" large wavelet coefficients. By a particular way of shrinking the small coefficients, one can "filter out" the noise. Such denoising problems occur in a complex problem setting which depends on the specific application so that several variants and customized versions of this basic idea are explained in these chapters. For example the role of the regularity of the wavelet basis, the basis being orthogonal or biorthogonal, separable or not, the exploitation of redundant transforms versus nonredundant transforms, etc., are all discussed. These denoising techniques are closely related to edge detection and contrast enhancement in images. Indeed, edges correspond to high frequencies, just like noise, but edges give large wavelet coefficients at different resolution levels so that they can be distinguished from noise. The decreasing of small coefficients and the increasing of large ones result in a better contrast of images such as radiographs.

Statistical methods are also an essential tool in these image processing techniques. For example, if the noise level is unknown, statistical techniques are introduced to estimate the noise threshold. Estimation of the local irregularity can reveal whether or not a pixel is

dominated by noise. Hence a mask is defined which can be put on the image and thus noise can be removed without losing the fine details of the true image.

The use of wavelet packages is also discussed. Usually, the wavelet transform decomposes a signal in a high- and a low-frequency component. The low-frequency part is again decomposed in two parts and so on. However, when splitting the high-frequency part as well, one obtains a redundant transform from which an optimal basis can be chosen to represent the image. All these techniques are illustrated in practical applications of X-ray computer tomography, magnetic resonance imaging, positron emission tomography, mammography, and many more.

Part III deals with biomedical signal processing. These are typically one-dimensional signals, in general time-varying, nonstationary, sometimes transient, and, again, corrupted by noise. We give a sample of the wavelet transform applications in this domain.

The excellent time localization property of wavelets is used to find several phenomena in a signal which occur at different frequencies and localize these events in time. For certain stochastic processes, such as action potentials or human heartbeat times, it is essential to estimate the fractal exponent of the process. Here again the wavelets are shown to outperform the Fourier transform. Furthermore, the continuous complex wavelet transform is used to analyze electrocardiograms. The modulus maxima and the  $\pm\pi/2$  phase crossing show the position of sharp signal transitions while modulus minima correspond to flat segments of the signal. In microvascular pulmonary pressure observations, two signals interfere. Here the signals are separated by using filtering techniques based on wavelets.

Part IV uses wavelets for mathematical models in biology. The multiresolution structure of the continuous wavelet transform corresponds to a natural human perception of sounds. Therefore wavelets are well suited to make auditory nerve models. To measure blood velocity, traditional methods are based on the Doppler effect when the movement of reflecting particles in the bloodstream are measured. It is illustrated here how the wideband wavelet transform gives a viable alternative. Event-related potentials are reactions of the brain to certain stimuli. Analysis of such signals is typically done by principal component analysis. However, it is shown that wavelets, due to their locality, allow to the analysis of such signals effectively. When using *a priori* information, the data can be drastically reduced. Also the structure of macromolecules can be deduced from a wavelet analysis of the energy function. Here the multiresolution of wavelets allows for the grouping of certain molecules. This technique can also be used to represent complex surfaces, like for example in computer tomography. This in a sense closes the circle in this wide variety of applications that are presented in this volume.

The book is of great importance for researchers working in medical or biological signal and image analysis. They will learn about wavelet alternatives for classical approaches. The wavelet researcher will certainly gain by learning about the particular problems posed by the applications of this particular, yet important field of wavelet based analysis.

Adhemar Bultheel

E-mail: Adhemar.Bultheel@cs.kuleuven.ac.be

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Th. M. Rassias and J. Šimša, *Finite Sums Decompositions in Mathematical Analysis*, Wiley, Chichester, 1995, vi + 172 pp.

This is a wonderful, well-written little book which was inspired by the simple question: which (scalar-valued) functions  $h = h(x, y)$  of two variables  $x$  and  $y$  have a representation in the form

$$h(x, y) = \sum_{i=1}^n f_i(x) g_i(y) \quad (1)$$