Exponential B-splines and the design of active contours and surfaces for biomedical image analysis

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Abstract

Snakes are effective tools for image segmentation. Within a 2D image, a snake is a 1D curve that evolves from an initial position, which is usually specified by a user, toward the boundary of an object. Within a 3D image, a snake is represented by a 2D surface. In the literature, these methods are also known as active contours or active surfaces.

Research has been fruitful in this area, and many snake variants have emerged. Among them, we are interested in the spline-based kind, where the curve or the surface is described continuously by some coefficients (a.k.a. control points) using basis functions. These snakes have become popular because it is possible for the user to interact with them, not only when specifying their initial position, but also during the segmentation process. This is often achieved by allowing the user to specify anchor points the curve or surface should go through.

Our interest is to characterize the spline-like integer-shift-invariant bases involved in the design of this kind of snakes. We prove that any compact-support function that reproduces a subspace of the exponential polynomials can be expressed as the convolution of an exponential B-spline with a compact-support distribution. As a direct consequence of this factorization theorem, we show that the minimal-support basis functions of that subspace are linear combinations of derivatives of exponential B-splines. These minimal-support basis functions form a natural multiscale hierarchy, which we utilize to design fast multiresolution algorithms and subdivision schemes for the representation of closed geometric curves and surfaces. This makes them attractive from a computational point of view.

Finally, we illustrate our scheme by building efficient active contours and surfaces capable of exactly reproducing ellipses in 2D and ellipsoids in 3D irrespective of their position and orientation.