Phase Retrieval: From Computational Imaging to Machine Learning

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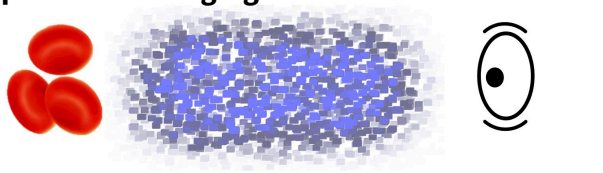
Phase Retrieval
Find $\boldsymbol{x}$ in
$\boldsymbol{y} = |\boldsymbol{A}\boldsymbol{x}|^2$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \left| \begin{bmatrix} a_{11} & \cdots & a_{1d} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nd} \end{bmatrix} \begin{bmatrix} x_1^* \\ \vdots \\ x_d^* \end{bmatrix} \right|^2$$

Computational imaging

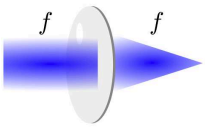


unknown object — sensing matrix — measurements

Fourier phase retrieval

$\mathbf{A} = \mathbf{F}$

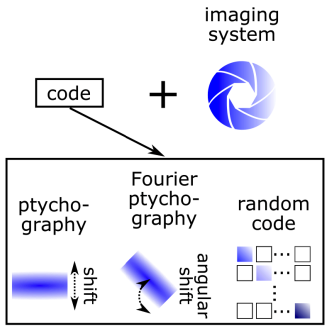
by a lens



by free-space propagation

Coded illumination

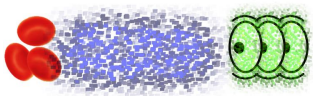
$\mathbf{A}_l = \mathbf{F}\mathbf{D}_l$



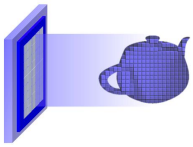
Coded detection

$\mathbf{A}_l = \mathbf{F}\mathbf{D}_l\mathbf{F}^{\mathrm{H}}$

phase diversity

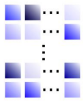


computer-generated holography



Random

by random projections



by propagation through complex media

Reconstruction algorithms

Alternating projections

First algorithms
Gerchberg-Saxton, Fienup, etc.
Still used today

Gradient-based optimization

Non-linear optimization
Simple and flexible
Few convergence guarantees

Convex relaxation

Rewrite the original equation
Closer to convex optimization
A few variants

Bayesian AMP

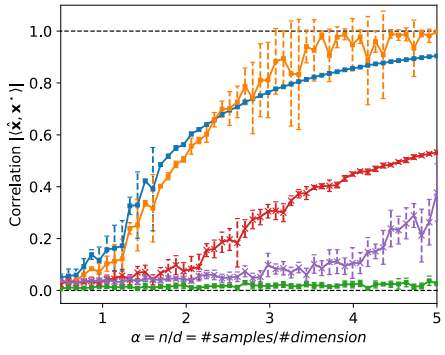
Based on Bayesian framework
Approximate Message Passing
Optimal among algorithms?

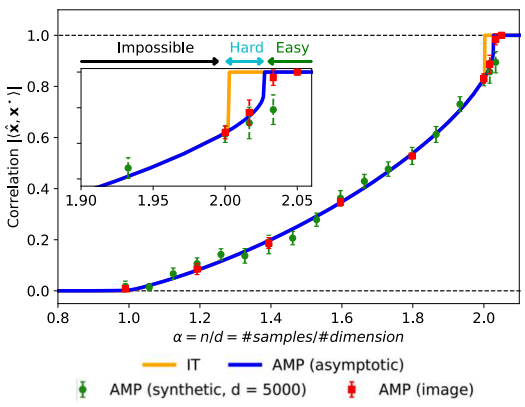
Spectral methods

Based on power iterations
Fast to compute
Provide good initial guess

Algorithm benchmark in the random setting

Correlation: 0 → random guess, 1 → perfect recovery





Machine Learning regularization

Direct inversion with NN

Train neural network to map measurements to reconstruction

Regularization using CNN denoisers

Denoising introduced at each iteration
RED, PnP, etc.

Generative models as priors

Train a GAN to generate images
Restrict search space to GAN outputs

Deep Image Prior

Use untrained CNN to generate estimate
Optimize weights to fit measurements

Others?

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