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Compression of ECG as a Signal with Finite Rate of Innovation

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***Abstract*— Compression of ECG (electrocardiogram) as a signal with finite rate of innovation (FRI) is proposed in this paper. By modelling the ECG signal as the sum of bandlimited and nonuniform linear spline which contains finite rate of innovation (FRI), sampling theory is applied to achieve effective compression and reconstruction of ECG signal. The simulation results show that the performance of the algorithm is quite satisfactory in preserving the diagnostic information as compared to the classical sampling scheme which uses the sinc interpolation.**

I. Introduction

Since many aspects of the physical condition of the human heart are reflected in the electrocardiogram(ECG) waveforms, it is important to record the patient's ECG for a long period of time for clinical diagnosis. Normally, a 24 hour or even longer duration recording is desirable for doctors to detect the human body's abnormalities or disorders, which can always be required in clinical applications such as telemedicine. This produces a large volume of ECG data everyday for storage and transmission. Reliable, accurate and efficient ECG data compression and reconstruction techniques are thus mandated.

In this paper, a new method of sampling and reconstruction of ECG data is proposed. The ECG signal is first modelled as the sum of a bandlimited signal and nonuniform linear spline which contains a finite rate of innovation. Then the signal is sampled at the rate of innovation in order to achieve compression of the ECG signal. Figure 1 gives the block diagram of the processing procedures.

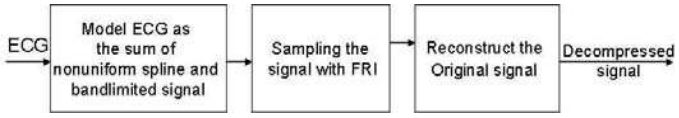

Fig. 1. Block diagram of the algorithm.

The rest of the paper is organized as follows: modelling of ECG signal as bandlimited plus nonuniform linear spline is given in Section 2, a brief review on sampling signals with FRI is given in Section 3, compression of ECG signal with finite rate is discussed in Section 4, finally experimental results on the proposed compression method are given in Section 5 and the conclusions in Section 6.

II. Modelling of ECG as the sum of bandlimited and nonuniform linear spline

Figure 2 shows the typical ECG signal with three indicated parts: P wave, QRS complex, and T wave. Since the ECG signal records the electrical potential at the electrode (or the potential difference between two electrodes) induced by the presence of time-varying electrical activity in cardiac muscle, by examining the shape of the ECG waveforms, a physician can obtain considerable insight about whether the contractions of the heart are occurring normally. So it is important to preserve the shape of the signal.

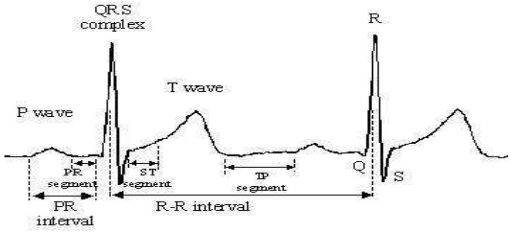

Fig. 2. Typical ECG signal.

From an engineering point of view, ECG signal is periodic, and Figure 2 shows one cycle of the signal. In order to preserve the diagnostic information in the ECG signal, the QRS complex has to be well preserved in the modelled signal. From the analysis of the QRS complex, it is easy to see that it can be modelled as nonuniform linear spline. By subtracting the QRS complex from the original signal, the remaining part can be modelled as a bandlimited signal.

Firstly, for delineating cycles, we define a cycle as the signal from one R-peak to the next. We use the technique reported in [2] for QRS detection. Once we get the R-peaks of the ECG signal, a P-QRS-T cycle can be extracted from the original signal. From the analysis of the morphology of the QRS complex, it can be approximated as a nonuniform linear spline. From the transition points we get from the QRS detection, linear interpolation is performed to get the linear approximation. The number of pieces is determined by the number of transition points. By subtracting the QRS complex from the original signal, the remaining part of the signal is analyzed. It is shown that the remaining part can be modelled as a bandlimited signal. The bandwidth of the bandlimited

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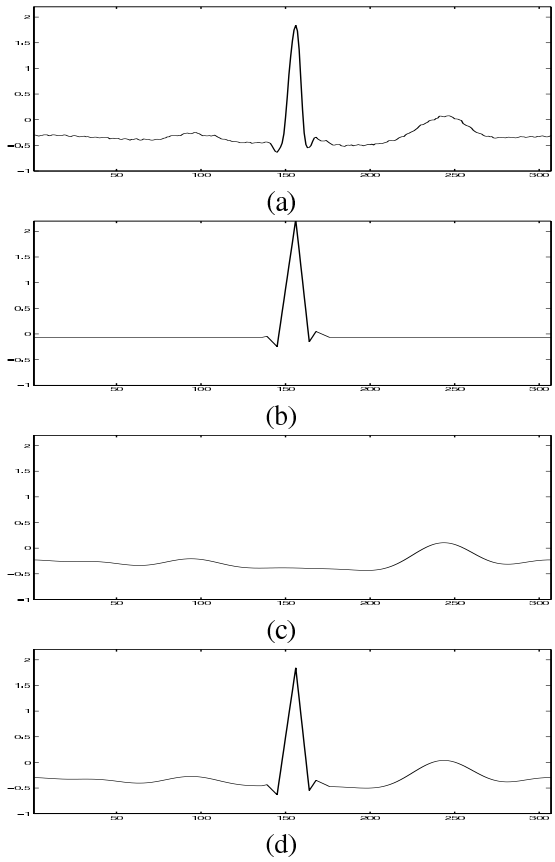

Fig. 3. Modelling of ECG signal as bandlimited plus nonuniform linear spline. (a) the original ECG signal; (b) the nonuniform spline approximation of the peak; (c) the bandlimited approximation of the remaining part of the signal; (d) the sum of the nonuniform linear spline and bandlimited signal. The vertical axis represents the amplitude and the horizontal axis represents the sample index.

approximation is determined by observing the distribution of the Fourier Spectrum. Thus, the sum of the nonuniform linear spline and bandlimited is an approximation of the original ECG signal. Figure 3 shows an example of approximating ECG signal as bandlimited plus nonuniform linear spline. We can easily see that the shape of the original signal is well preserved in the approximated one.

III. Review on sampling signals with finite rate of innovation

Consider classes of signals which have a finite number of degrees of freedom per unit of time, and call this number the rate of innovation. Examples of signals with a finite rate of innovation include streams of Diracs, nonuniform splines and piecewise polynomials. Even though these signals are not bandlimited, it is shown in [3], [4], [5] that they can be sampled uniformly at (or above) the rate of innovation using an appropriate kernel, and then can be perfectly reconstructed.

A. Periodic Stream of Diracs

Consider a stream of K Diracs periodized with period τ, $x(t) = \sum_{n\in\mathbb{Z}} c_n\,\delta(t-t_n)$ where $t_{n+K} = t_n + \tau$ and $c_{n+K} = c_n, \forall n \in \mathbb{Z}$. This signal has $2K$ degrees of freedom per period (K from the locations and K from the weights), thus the rate of innovation is

$$\rho = \frac{2K}{\tau}. \tag{1}$$

The algorithm for sampling and reconstruction of periodic stream of Diracs is described as follows:

1) Calculate the sample values
$$y_n = < h_B(t-nT), x(t) >, \quad n = 0,\ldots,N-1, \tag{2}$$
where $h_B(t) = B\mathrm{sinc}(Bt)$, $B = \rho$ and T is a divisor of τ, thus $N = \tau/T$.
2) Find $2K$ contiguous spectral values of $x(t)$ from the sample values
$$y_n = \sum_{m\in\mathbb{Z}} X[m] < h_B(t-nT), e^{i\frac{2\pi mt}{\tau}} > \tag{3}$$
$$= \sum_{m\in\mathbb{Z}} X[m]\, H_B(\frac{2\pi m}{\tau})\, e^{i\frac{2\pi mnT}{\tau}} \tag{4}$$
$$= \sum_{m=-K}^{K} X[m]\, e^{i\frac{2\pi mnT}{\tau}} \tag{5}$$
where $H_B(\omega) = \mathrm{Rect}(\frac{\omega}{2\pi B})$ is the Fourier transform of $h_B(t)$. The $2K+1$ contiguous coefficients of $X[m]$ can be obtained by solving this system of equations [1].
3) Find the annihilating filter coefficients $A[m]$. Given $X[m], m = -K,\ldots,K$, solve the following Toeplitz system of equations for $A[m], m = 1,\ldots,K$,
$$A[m] * X[m] = 0. \tag{6}$$
4) Find the locations t_k. Given the coefficients $1, A[1],\ldots,A[K]$, we factor its $z-$ transform into its roots
$$A(z) = \prod_{k=0}^{K-1}(1 - u_k\, z^{-1}) \tag{7}$$
where $u_k = e^{-i\frac{2\pi t_k}{\tau}}$, which leads to the K locations $\{t_k\}_{k=0}^{K-1}$.
5) Find the weights c_k. Given the locations $\{t_k\}_{k=0}^{K-1}$, we have K values of $X[m] = \frac{1}{\tau}\sum_{k=0}^{K-1} c_k u_k^m$, which is a Vandermonde system of equations, and since the t_k's are distinct, this leads to a unique solution of c_k's.

That is, by sampling the periodic stream of Diracs at the rate of innovation, the original Diracs can be perfectly reconstructed.

B. Periodic Nonuniform splines

A signal $x(t)$ is a periodic nonuniform spline of degree R with knots at $\{t_k\}_{k=0}^{K-1} \in [0,\tau]$ if and only if its $(R+1)$th derivative is a periodic stream of K weighted Diracs $x^{(R+1)}(t) = \sum_{k=0}^{K-1} c_k\,\delta(t-t_k)$. Thus the rate of innovation is

$$\rho = \frac{2K}{\tau}. \tag{8}$$

[1]Note that $N = \tau/T$, y_n corresponds to the IDFT of $X[m]$.

The algorithm for sampling and reconstruction of periodic nonuniform splines is described as follows:

1) Calculate the sample values at the rate of innovation, that is, take $B = \rho$ and T as a divisor of τ, then $N = \tau/T$.
2) Find $2K$ contiguous spectral values of $x(t)$ from the sample values.
3) Find $2K$ corresponding contiguous spectral values of the stream of Diracs, we have

$$X^{(R+1)}[m] = (i2\pi m/\tau)^{R+1} X[m]. \quad (9)$$

4) Determine the locations and weights of the Diracs $x^{(R+1)}(t)$ using the annihilating filter method.
5) Get the original splines by integrating $R+1$ times the Diracs.

IV. Compression and reconstruction of ECG signal

In this stage, the sampling theorem for signals with finite rate of innovation will be applied to compress and reconstruct the ECG signal. According to the method in Section III, given the number of pieces in nonuniform linear spline and the bandwidth of the bandlimited signal, the sum of these two signals can be perfectly reconstructed.

We approximate the QRS complex as a nonuniform linear spline with K pieces, and the rest of the signal is approximated as a bandlimited signal with bandwidth of L, where the period of the original signal is τ. Then the modelled signal x is defined by

$$x(t) = x_{BL}(t) + x_{NS}(t), \quad (10)$$

with corresponding CTFS coefficients defined by

$$X[m] = \begin{cases} X_{BL}[m] + X_{NS}[m] & \text{if } m \in [-L, L] \\ X_{NS}[m] & \text{if } m \notin [-L, L]. \end{cases}$$

The rate of innovation for the modelled signal is $\rho = (2L + 1 + 2K)/\tau$, where τ is the period.

Consider the nonuniform spline of degree one with K pieces, it can be recovered from $2K$ contiguous frequency values $X_{NS}[m]$. Therefore it is sufficient to take $2K$ CTFS coefficients outside of the band $[-L, L]$. Once $x_{NS}(t)$ is reconstructed, the CTFS of the bandlimited are obtained by subtracting $X_{NS}[m]$ from $X[m]$ for $m \in [-L, L]$.

The algorithm of sampling and reconstruction of ECG signal is described as follows:

1) Calculate the sample values.

$$y_n = < h_B(t - nT), x(t) >, \quad n = 0, \ldots, N-1, \quad (11)$$

where $B = 2(L+2K)/\tau$, note that here we are sampling with $2K/\tau$ above the rate of innovation. Take T as a divisor of τ, then $N = \tau/T$.
2) Find $2(2K+L)$ contiguous spectral values of $x(t)$ from the sample values.
3) Take $2K$ contiguous values of $X_{NS}[m] = X[m], m \in [L+1, L+2K]$.
4) Find $2K$ contiguous spectral values of the stream of Diracs,

$$X_{NS}^{(R+1)}[m] = (i2\pi m/\tau)^{R+1} X_{NS}[m], m \in [L+1, L+2K]. \quad (12)$$

5) Determine the locations and weights of the Diracs $x^{(R+1)}(t)$ using annihilating filter method.
6) Get the original spline $x_{NS}(t)$ by integrating $R+1$ times the Diracs, thus we have $X_{NS}[m]$, $m \in \mathbb{Z}$.
7) Find the bandlimited signal $x_{BL}(t)$. Once we get $X_{NS}[m]$, we have

$$X_{BL}[m] = X[m] - X_{NS}[m], \quad m \in [-L, L], \quad (13)$$

and $x_{BL}(t)$ is thus recovered.
8) The original signal is the sum of the bandlimited signal and nonuniform linear spline.

V. Experimental Results

The proposed method was tested on ECG data from MIT/BIH Arrhythmia Database [1]. Figures 4, 5 and 6 give the original, reconstructed signal using sampling signal with FRI and sinc kernel, respectively. It is obvious to see that sampling the signal with finite rate of innovation performs better in preserving the morphological and diagnostic information in ECG signal. Table I gives the reconstruction performance figures for four different experimental signals. We get smaller reconstruction error by comparing our method with the classic sampling using sinc interpolation. The Compression Ratio (CR) is defined as the ratio of the number of samples in the original ECG signal and $2K + L$. Table II gives the compression ratio for some of the signals from the database, the ratios are quite satisfactory.

TABLE I
Comparison of reconstruction error of sampling with FRI and sinc kernel

Record	103	115	116	123
FRI	8.4%	7.7%	3.4%	2.8%
sinc	11.7%	12.7%	3.5%	5.9%

TABLE II
Compression Ratio Performance on Signals form MIT-BIH Arrythmia Database

Record	103	115	116	123
K	8	8	8	8
L	6	4	4	9
CR	14.0	17.3	12.5	19.1

VI. conclusions

In this paper, a novel algorithm for ECG data sampling and reconstruction has been proposed. Firstly, we model ECG signal as the sum of bandlimited signal and nonuniform linear spline. Given the $2K + L$ coefficients of $X[m]$, the

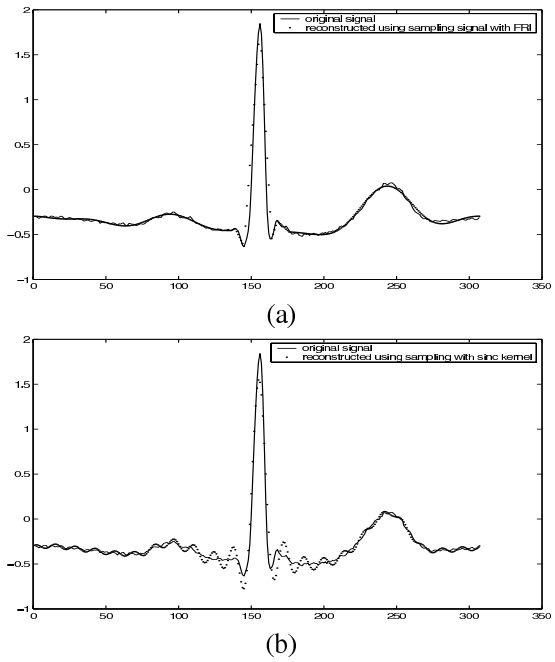

Fig. 4. Results on record 103 from the MIT-BIH Arrythmia Database: (a) reconstruction of ECG using sampling signal with finite rate of innovation, with reconstruction error of 19%; (b) reconstruction of ECG using sampling with sinc kernel, with reconstruction error of 17%. The vertical axis represents the amplitude and the horizontal axis represents the sample index.

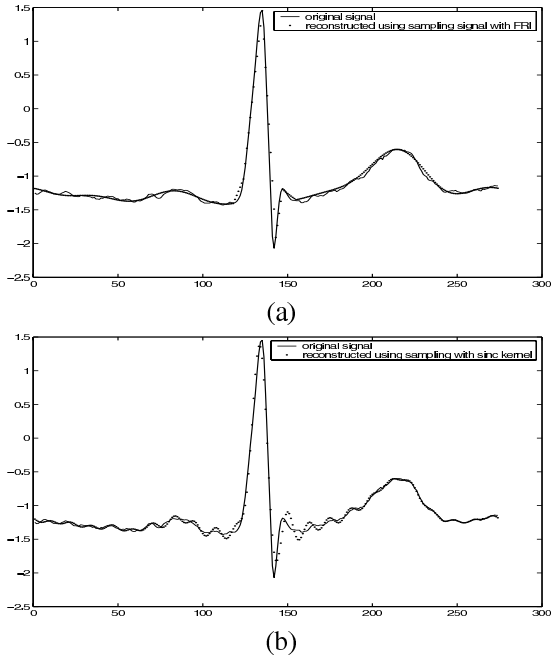

Fig. 5. Results on record 116 from the MIT-BIH Arrythmia Database: (a) reconstruction of ECG using sampling signal with finite rate of innovation, with reconstruction error of 7%; (b) reconstruction of ECG using sampling with sinc kernel, with reconstruction error of 6%. The vertical axis represents the amplitude and the horizontal axis represents the sample index.

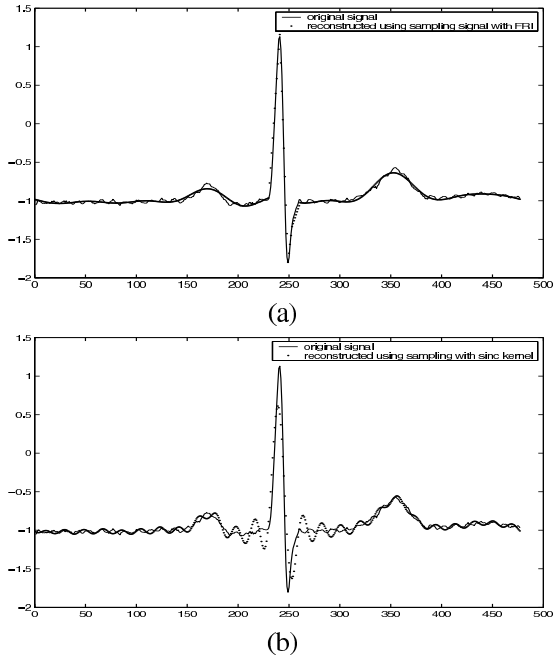

Fig. 6. Results on record 123 from the MIT-BIH Arrythmia Database: (a) reconstruction of ECG using sampling signal with finite rate of innovation, with reconstruction error of 5%; (b) reconstruction of ECG using sampling with sinc kernel, with reconstruction error of 11%. The vertical axis represents the amplitude and the horizontal axis represents the sample index.

nonuniform spline is recovered from $2K$ of contiguous Fourier series coefficients outside the band of $m \in [-L, L]$. The Fourier series coefficients of the bandlimited signal, X_{BL}, is obtained by subtracting X_{NS} from $X[m]$ inside the band of $m \in [-L, L]$. Generally speaking, the original signal can be reconstructed given certain number of Fourier series coefficients. By comparing the simulation results with the ones achieved by classic sinc interpolation, it is shown that the performance of the proposed one is much better than the latter, especially in preserving the morphological information of the signal, which is an important factor in biomedical signal processing.

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