STATISTICAL ANALYSIS OF IMAGE DIFFERENCES BY WAVELET DECOMPOSITION

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Abstract
The wavelet transform was studied for the analysis of glucose utilization differences between subject groups shown in PET images. To strengthen statistical inference, it was of particular interest investigating the tradeoff between signal localization and image decomposition into uncorrelated components. This tradeoff is governed by wavelet regularity, and was found to be best for third-order orthogonal spline wavelets. Only about 1.6% of the components were statistically different ($p<.05$) from noise, constituting a sufficient set to synthesize local image differences by the inverse wavelet transform.

1. Introduction

Quantitative neuroimaging by positron emission tomography (PET) is a major research tool to study *in vivo* dynamic processes occurring in discrete brain regions. To investigate functional differences between groups of subjects, the brain is usually subdivided into regions of interest (ROIs), which are then averaged within groups and analyzed by univariate t-tests. However, this approach is questionable due to correlations among ROIs introduced by the imaging physics. One may avoid the problems arising from spatial correlations by conducting the analysis in the Fourier transform domain, where the power estimates at different spatial frequencies are (asymptotically) uncorrelated. While this permits rigorous testing as to whether any differences between subject groups exist, the spatial location of these differences cannot be specified satisfactorily due to the mathematical property that a function cannot have finite support in both the original and the Fourier domain (uncertainty principle). Since good localization in the image domain and decomposition into uncorrelated components cannot be achieved simultaneously, the establishment of approximate methods that strike an acceptable compromise between these two conflicting constraints is of practical importance. Hence, we investigated the use of the wavelet transform as a possible method for obtaining both adequate localization and approximate decorrelation of image components.

2. Methods

The PET images for this study represented cerebral glucose utilization rates, and were obtained from 10 patients with alcoholic organic mental disorders and 7 normal volunteers. For each subject, 21 slices with 128x128 pixels (pixel size=2 mm) were acquired with the NeuroPET scanner, providing transverse and axial spatial resolution of 7 and 11.5 mm (FWHM), respectively, with an interslice separation of 4 mm. Clinicians selected comparable slices across individuals for analysis. The images were matched to a common standard image at each slice level by global affine transformations involving translation, rotation and anisotropic scaling, and then averaged for each group separately. Subtraction of the two group-averages produced images of the functional differences between the alcoholics and the normal volunteers. The group variance at each pixel location was also obtained to construct a null image for each comparison, which consisted of nonstationary noise with mean zero and variance of the gray levels equal to that in corresponding group-difference images.

In this study, we investigated the use of orthogonal spline wavelets (Lemarié-Battle), which permit the decomposition of an image into orthogonal components (wavelets). These components are obtained by dilation and translation of a basic wavelet, and their amplitudes are called wavelet coefficients. While the wavelet decomposition base is chosen to be orthogonal for statistical reasons, orthogonality of the base does not imply absence of correlations among the wavelet coefficients. Decorrelation can be achieved only to the extent that the Fourier transforms of the wavelets at different scales do not overlap, i.e. approximate ideal bandpass filters. However, the better this approximation, the larger the support required for the implementation of the wavelets (uncertainty principle) and thus, loss of localization. It is shown that the correlation between wavelet coefficients across scales in the ratio $r > 1$ decays as $\exp[-(p+3/2)\ln r]$, where p is the highest order of vanishing moments (regularity) of the basic wavelet. Hence, for the spline wavelet family, the consequences of the tradeoff between wavelet support and cross-correlation decay can be easily investigated by changing the order of the spline polynomials.

The wavelet transforms were computed by the iterative method of Mallat [1], yielding wavelet coefficients at each level in a resolution pyramid, where at each successive level the image resolution is decreased by factor 2. Five iteration steps were applied, and the wavelet coefficients at each scale subjected to statistical analyses.

3. Results

Omnibus tests were first performed at each resolution scale to test the hypothesis whether at that scale the wavelet coefficients of the test image (group-average difference image) differed significantly from those of the null image. The variance ratio of the wavelet coefficients for the test and null images was compared against the

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$F_{v,v}$ -distribution, where $v=(128/2^k \times 128/2^k)$ degrees of freedom at scale, k. The tensor product implementation of the two-dimensional wavelet transform yielded 3 sets of coefficients (highpass bands in only the x-direction, only the y-direction, and simultaneously in the x- and y-direction) at each scale, requiring for 5 resolution levels a total of 15 tests. Hence, the significance level was set at $\alpha=0.05/15=0.003$ to maintain type I error protection.

Table 1. Ratios of wavelet coefficient variances for PET-difference image vs. null image

Scale [a]		Spline Wavelet Order 0	1	3	5	F-crit [b]
	H	.25	.20	.20	.20	
1	V	.18	.13	.12	.12	1.09
	HV	.08	.07	.06	.06	
	H	.90	.62	.54	.53	
2	V	.58	.42	.38	.37	1.18
	HV	.30	.26	.27	.28	
	H	4.87	5.51	5.49	5.61	
3	V	2.53	3.75	3.45	3.21	1.40
	HV	1.71	1.62	1.59	1.57	
	H	23.23	41.81	41.40	40.87	
4	V	20.00	12.12	13.08	13.17	1.99
	HV	6.61	9.22	10.12	10.21	
	H	97.95	127.91	135.21	138.03	
5	V	42.06	77.15	70.03	61.10	4.19
	HV	19.23	21.25	18.04	18.65	

[a] H: horizontal, V: vertical,
HV: horizontal & vertical decomposition

[b] p = 0.05/15 = 0.0033 for Bonferroni adjustment

This analysis is summarized in Table 1 and shows that irrespective of the spline orders, the wavelet coefficients at the 2 highest resolution levels could be discarded out of hand as encoding noise only. Hence, the highest resolution relevant was $2^2=4$ pixels, or 8 mm, consistent with the transverse resolution of the scanner of 7 mm. With the elimination of the 2 highest resolution levels, only $(2^5 \times 2^5) - 4 \times 4 = 1008$ coefficients or 6.15% of the total coefficients were needed to be tested individually for significance, resulting in 209 *(1.28%)*, 238 *(1.45%)*, 259 *(1.58%)*, and 265 *(1.62%)* coefficients for spline wavelets of order 0, 1, 3, and 5, respectively. The smaller number of significant coefficients for lower-order wavelets reflects the loss of signal power at low spatial frequencies due to spectral leakage into higher-resolution channels.

The impact of wavelet order on spectral leakage was evaluated specifically by the correlation between wavelet coefficients across different scales. For a scale ratio of r=2, the measured correlations for spline orders 0 and 1 were statistically significant ($p<0.05$), while for r=4 only zero-order splines showed significant correlations.

The quality of the difference images reconstructed by inverse wavelet transform of only the significant wavelet coefficients was compared visually for splines of order 0, 1, 3 and 5. For the zero-order synthesis the images had an objectionable "blocky" appearance, some linear streaking artifacts were visible for first-order splines, and little differences could be discerned between third- and fifth-order spline reconstructions. Since the removal of all image components at the two highest resolution scales corresponds to spatial lowpass filtering, the reconstructed images appeared less noisy than the original difference images, with regional differences not broken up into single-pixel regions.

Discussion

A considerable reduction of dimensionality was attained by applying global tests of the wavelet coefficients at each resolution level. To the extent that the coefficients at different scales are uncorrelated, these tests are independent, rendering the Bonferroni adjustment for multiple testing less conservative.

Overviewing the methods developed from the aspect of signal processing, it is desirable to select an image representation that maximizes the signal-to-noise ratio (SNR) in the transformed domain. In our application, most signal power is concentrated at the coarsest resolution levels. Accordingly, we should expect wavelet transforms with better bandpass characteristics (i.e., larger regularity indices) to make better use of the SNR available at low-resolution channels. In practice, this choice is restricted by the fact that higher-order wavelets are less localized. Based on our measurements of the correlations across different resolution levels and the quality of the image reconstruction from significant components only , there was no compelling reason to choose wavelets of orders higher than 3.

In conclusion, for the decomposition of PET images splines of order 3 are sufficient. Group-difference images can be resynthesized from only a very low fraction (≈1.6%) of statistically significant wavelet coefficients, resulting in relatively uniform, noise-free regions of glucose utilization differences.

References

[1] S.G. Mallat, "A theory of multiresolution signal decomposition: the wavelet representation", *IEEE Trans. Patt.Anal. Machine Intell.*, vol. 11, pp. 674-693, 1989.