OPTIMAL WIENER FILTERING FOR fMRI IMAGES WITH POLYHARMONIC SMOOTHING SPLINES

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INTRODUCTION: Motivated by the fractal-like behavior of fMRI images[1] (and other images as well [2]), we propose a smoothing technique which uses a regularization functional that is a fractional iterate of the Laplacian.

This type of functional was introduced by Duchon in the context of radial basis functions (RBFs). We solve it using non-separable fractional polyharmonic B-splines[3].

We show a way of choosing the order of differentiation s, and prove that our algorithm is equivalent to the optimal discretization of the continuous-time Wiener filter for fractal-like signals (with a $O(|\omega|^{-s})$ spectral decay).

METHODS: We prove that the solution of Duchon's regularized smoothing problem can be obtained by using polyharmonic splines. The smoothing is performed in the Fourier domain by filtering, yielding a fast and simple algorithm:

$$\hat{f}(\boldsymbol{\omega}) = \frac{B_{2s}\left(e^{j\boldsymbol{\omega}}\right)}{B_{2s}\left(e^{j\boldsymbol{\omega}}\right) + \lambda \left\|2\sin\left(\boldsymbol{\omega}/2\right)\right\|^{2s}}\, \hat{g}(\boldsymbol{\omega}), \quad (1)$$

where $\hat{g}(\omega)$ is the Fourier transform of the input image and B_{2s} is the DFT of the autocorrelation sequence of the discrete polyharmonic B-splines of order s.

RESULTS: We applied our approach to the denoising of MRI T2* data. A reference slice is shown in the inset. To test the algorithm, we added noise to it, and report the resulting SNR for different values of s in Fig. 1.

Figure 2 shows the radial frequency response of our image, and the regressed fractal model (on a log-log scale). Note that the value of s provided by this analysis corresponds to optimal one in Fig 1. In Fig 3 we compare our method with a reference Wiener filter (which uses the a priori knowledge of the spectrum of the noise-free signal) for different noise levels, and demonstrate their equivalence experimentally.

DISCUSSION & CONCLUSIONS: We found a viable way to get the optimal discretization of the continuous-time Wiener filter in multiple dimensions, which is the optimal linear solution possible, for fractal-like signals. Even though our approach has only two parameters (which can be estimated from the noisy data), it produces results that are very close to the ideal Wiener filter, which is not available in practice.

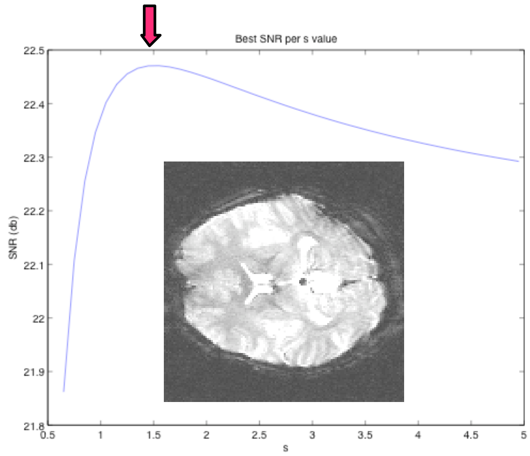

Fig. 1: Original image and result SNR per s.

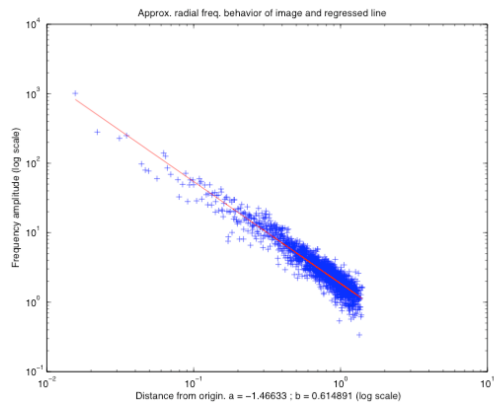

Fig. 2: Radial frequency profile and regressed fractal model.

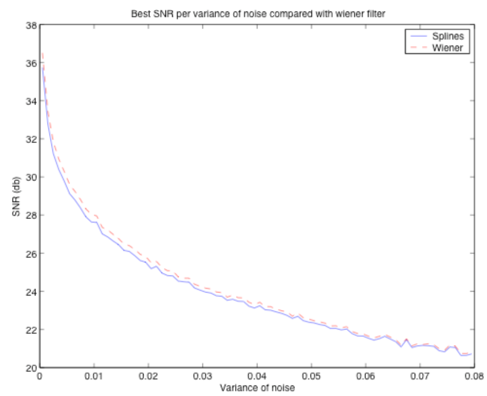

Fig. 3:Comparison with optimal Wiener filter.

REFERENCES: [1]E. Zarahn *et. al.* (1997) *NeuroImage* **5**:179-197. [2]A.P Pentland (1984) *IEEE Trans. PAMI* **6**:661-674. [3]C. Rabut (1992) *Numer. Algo.* **2**:39-62.