SPLINES: A PERFECT FIT FOR SIGNAL PROCESSING

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Splines, which were invented by Schoenberg more than fifty years ago [1], constitute an elegant framework for dealing with interpolation and discretization problems. They are widely used in computer-aided design and computer graphics, but have been neglected in signal and image processing applications, mostly as a consequence of what I call the “bad press” phenomenon. Thanks to some recent research efforts in signal processing and wavelet-related techniques, the virtues of splines have been revived in our community [2]—there is now compelling evidence (several independent studies [3-5]) that splines offer the best cost-performance tradeoff among available interpolation methods.

In this talk, I will argue that the spline representation is ideally suited for all processing tasks that require a continuous model of signals or images. I will show that most forms of spline fitting (interpolation, least squares approximation, smoothing splines) can be performed most efficiently using recursive digital filters. I will discuss the connection between splines and Shannon’s sampling theory. I will also look at their multiresolution properties which make them prime candidates for constructing wavelet bases and computing image pyramids. I will provide multiple illustrations of their use in image processing; these include zooming and visualization, geometric transformation, registration, contour detection, as well as snakes and contour modeling.

References

Spline basics:

[1] I.J. Schoenberg, "Contribution to the problem of approximation of equidistant data by analytic functions," *Quart. Appl. Math.*, vol. 4, pp. 45-99, 112-141, 1946.

[2] M. Unser, "Splines: A perfect fit for signal and image processing," *IEEE Signal Processing Magazine*, vol. 16, no. 6, pp. 22-38, 1999.

Comparison of splines and other interpolation methods:

[3] T.M. Lehmann, C. Gönner and K. Spitzer, "Survey: Interpolation methods in medical image processing," *IEEE Transactions on Medical Imaging*, vol. 18, no. 11, pp. 1049-1075, 1999.

[4] P. Thévenaz, T. Blu and M. Unser, "Image interpolation and resampling," in *Handbook of Medical Image Processing*, in press.

[5] E.H.W. Meijering, "Spline interpolation in medical imaging: Comparison with other convolution-based approaches," in Proc. *European Signal Processing Conference*, Tampere, Finland, September 5-8, 2000, in press.