A Guided Tour of Splines for Medical Imaging

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Abstract

Splines, which were invented by Schoenberg more than fifty years ago, constitute an elegant framework for dealing with interpolation and discretization problems. Our purpose in this talk is to motivate their use in medical imaging, emphasizing their ease of use, as well as their fundamental properties. In particular, we will describe efficient digital filtering algorithms for the interpolation and spline-based processing of signals and images. We will show that splines are intimately linked to differentials and identify B-splines as the exact mathematical translators between the discrete and continuous versions of the (scale-invariant) operator. This partly explains why these functions play such a fundamental role in wavelet theory. Splines may also be justified on variational and/or statistical grounds; e.g., they provide Wiener (i.e, MMSE) estimators for fractal processes such as fractional Brownian motion. We will illustrate spline processing with applications in biomedical imaging where its impact has been the greatest so far. Specific tasks include high-quality interpolation, snakes, and various types of image registration. There is now compelling evidence (several independent studies in medical imaging) that splines offer the best cost- performance tradeoff among available interpolation methods.

Software and demos are available at: `http://bigwww.epfl.ch/`

Biography

Michael Unser is Professor and Director of EPFL's Biomedical Imaging Group, Lausanne, Switzerland. His main research area is biomedical image processing. He has a strong interest in sampling theories, multiresolution algorithms, wavelets, and the use of splines for image processing. He has published over 150 journal papers on those topics, and is one of ISI's Highly Cited authors in Engineering (http://isihighlycited.com).

From 1985 to 1997, he was with the Biomedical Engineering and Instrumentation Program, National Institutes of Health, Bethesda USA, conducting research on bioimaging and heading the Image Processing Group.

Dr. Unser is a fellow of the IEEE and the recipient of three Best Paper Awards from the IEEE Signal Processing Society.

Selected reading list for learning more about splines

Spline tutorial:

- M. Unser, "Splines: A Perfect Fit for Signal and Image Processing," IEEE Signal Processing Magazine, vol. 16, no. 6, pp.22-38, 1999.

Splines and recursive filtering algorithms:

- M. Unser, A. Aldroubi, M. Eden, "B-Spline Signal Processing: Part II-Efficient Design and Applications," IEEE Transactions on Signal Processing, vol. 41, no. 2, pp. 834-848, February 1993.

Splines and sampling theory:

- M. Unser, "Sampling-50 Years After Shannon," Proceedings of the IEEE, vol. 88, no. 4, pp. 569-587, April 2000.

Splines and approximation theory:

- T. Blu, M. Unser, "Quantitative Fourier Analysis of Approximation Techniques: Part I-Interpolators and Projectors," IEEE Transactions on Signal Processing, vol. 47, no. 10, pp. 2783-2795, October 1999.

Splines and linear systems theory:

– M. Unser, "Cardinal Exponential Splines: Part II-Think Analog, Act Digital," IEEE Trans. Signal Processing, vol. 53, no. 4, pp.1425-1438, April 2005.

Splines and wavelet theory:

– M. Unser, T. Blu, "Wavelet Theory Demystified," IEEE Trans. Signal Processing, vol. 51, no. 2, pp. 470-483, 2003.

Splines, operators and fractals:

– M. Unser, T. Blu, "Self-Similarity: Part I-Splines and Operators," IEEE Transactions on Signal Processing, vol. 55, no. 4, pp.1352-1363, April 2007.

– T. Blu, M. Unser, "Self-Similarity: Part II-Optimal Estimation of Fractal Processes," IEEE Transactions on Signal Processing, vol. 55, no. 4, pp. 1364-1378, April 2007.

Splines and stochastic processes:

– M. Unser, T. Blu, "Generalized Smoothing Splines and the Optimal Discretization of the Wiener Filter," IEEE Transactions on Signal Processing, vol. 53, no. 6, pp. 2146-2159, June 2005.