Wavelets and Advanced Biomedical Imaging

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***Abstract*—Our purpose in this talk is to advocate the use of wavelets for advanced bioimaging. We start with a short tutorial on wavelet bases, emphasizing the fact that they provide a concise multiresolution representation of images and that they can be computed most efficiently. We then discuss a simple but remarkably effective image-denoising procedure that essentially amounts to discarding small wavelet coefficients (soft-thresholding); we show that this type of algorithm is the solution of a variational problem that promotes sparse solutions. We argue that the underlying principle of wavelet regularization is a powerful concept that can be used advantageously in a variety of inverse image-reconstruction problems, including MRI and computed tomography. We illustrate our point by presenting a novel wavelet-based deconvolution algorithm for 3-D fluorescence microscopy, as well as some preliminary results for dynamic PET reconstruction. We will also discuss wavelet techniques for the analysis of functional MRI data and optical microscopy (extended depth of field).**

***Index Terms*—Wavelets, sparsity, denoising, soft-thresholding, regularization, deconvolution, image reconstruction**

I. Introduction

Wavelets are a powerful way of decomposing signals or images into their elementary constituents across scales (multiresolution decomposition). They provide a one-to-one representation (orthogonal transform) in very much the same way as the Fourier transform does, except that the basis functions are localized in both space (or time) and frequency. Wavelets have many remarkable properties and are extremely versatile. It is therefore no surprise that they have been applied to a variety of problems in biomedical imaging, often with good success [1], [2]. The property that is often emphasized in current applications is their ability to yield sparse representations of piecewise-smooth signals and images.

Rather than providing a partial account of the plenary presentation whose content we just summarized above, we have chosen to give in this brief some pointers to the relevant literature, together with some personal remarks.

1) Introduction to wavelets: Among the many books available, we recommend the ones who were written by the pioneers of the field; in particular, Stéphane Mallat [3] and Ingrid Daubechies [4]. The reader who wants to dive into the more theoretical aspects of wavelets may be interested in [5], which has the advantage of being self-contained. At the other end of the scale of complexity, we mention a low-level introduction in the form of a humorous dialog between father and son [6].

2) Wavelet denoising by soft-thresholding: A not-so-well-known fact is that this approach was pioneered by Weaver *et al.* in the context of magnetic resonance imaging [7]. The technique is usually attributed to Donoho who gave a rigorous statistical justification and was very proactive in terms of promotion and software diffusion [8], [9]. Interestingly, it took quite a few years until Chambolle *et al.* finally showed that the algorithm solves a penalized least-squares problem with a wavelet-domain ℓ_1-regularization [10]. Another important point, which is also central to the field of "compressed sensing," is that the ℓ_1-norm is a good (convex) proxy for the ℓ_0-norm—the latter simply counts the number of non-zero coefficients. It is also known that one can solve the basic denoising/approximation problem with an ℓ_0 penalty in any orthonormal basis by the straightforward application of a hard threshold in the transformed domain; unfortunately, this type of ℓ_0 result is much harder to generalize because the corresponding criterion is non-convex.

It should also be mentioned that better performance for image denoising can be obtained by adapting the wavelet-domain non-linearity to the type of noise and to the information content of the image [11], [12], [13].

3) Image reconstruction by iterative thresholding: This powerful algorithm was discovered by engineers who noted that they could improve traditional gradient-based iterative image-reconstruction schemes by inserting a wavelet-denoising step in the feedback correction loop (denoising of the residual). Figueireido and Nowak were among the first to provide a rigorous statistical justification of the method within the EM framework [14]. Daubechies, De Frise and De Mol took a general deterministic variational point of view (ℓ_1-regularized least-squares inverse problem) and were able to prove the convergence of the algorithm (often termed D^3) under relatively mild conditions [15]. While several variants and generalization of the technique are now available, researchers are still working hard on finding ways to accelerate the convergence. The increasing interest in this type of convex optimization problem is also a consequence of the popularity of "compressed sensing".

4) Wavelet regularization in biomedical imaging: A challenging application is 3-D fluorescence deconvolution, mainly because of the huge data size [16]. Making the wavelet approach practical requires the development of an effective divide-and-conquer acceleration strategy.

While wavelet regularization is usually applied in the space domain, it can be extended to the time domain for dynamic imaging. An interesting application is the reconstruction of dynamic PET which can be improved by combining spatial wavelets with exponential splines that are specially tailored to the time-domain characteristics of the activity curves [17].

Other applications that are briefly reviewed are: wavelet-based image fusion for extended depth of focus [18], and a general framework for the statistical analysis of functional imaging data [19], [20], [21].

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