# Gaussian vs. sparse stochastic processes: a unifying spline-based perspective

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## Abstract

The main point that is made in this talk is that splines and stochastic processes can be linked by some common operator formalism.

The stochastic processes of interest are generalized functions that are solutions of (possibly unstable) stochastic differential equations (SDE). They constitute an extension of the classical Lévy processes on the one hand, and the Gaussian stationary processes on the other. Fractal processes such fractional Brownian motion are also part of the family. These generalized processes are described by a general innovation model that is specified by: 1) a whitening operator (e.g., fractional derivative), which shapes their second-order moments, and 2) a Lévy exponent $f(\omega)$, which controls the sparsity of the (non-Gaussian) innovations (white Lévy noise).

We show how the underlying SDEs can be solved by defining some appropriate inverse operator subject to some (adjoint) $L_p$ boundedness constraint. The method results in a complete characterization of the processes via their characteristic form (the infinite-dimensional counterpart of the characteristic function). This allows us to prove that they admit a sparse representation in a wavelet bases. We provide evidence that wavelets allow for a better $N$ term approximation than the classical Karhunen-Loève transform (KLT), except in the Gaussian case. We also highlight a fundamental connection between the construction of maximally localized basis functions (B-splines) and statistical decoupling (including a link with the Markov property).