# Towards a theory of sparse stochastic processes, or when Paul Lévy joins forces with Nobert Wiener

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ABSTRACT

The current formulations of compressed sensing and sparse signal recovery are based on solid variational principles, but they are fundamentally deterministic. By drawing on the analogy with the classical theory of signal processing, it is likely that further progress may be achieved by adopting a statistical (or estimation theoretic) point of view. Here, we shall argue that Paul Lévy (1886-1971), who was one of the very early proponents of Haar wavelets, was in advance over his time, once more. He is the originator of the Lévy-Khinchine formula, which happens to be the perfect (non-Gaussian) ingredient to support a continuous-domain theory of sparse stochastic processes.

Specifically, we shall present an extended class of signal models that are ruled by stochastic differential equations (SDEs) driven by white Lévy noise. Lévy noise is a highly singular mathematical entity that can be interpreted as the weak derivative of a Lévy process. A special case is Gaussian white noise which is the weak derivative of the Wiener process (a.k.a. Brownian motion). When the excitation (or innovation) is Gaussian, the proposed model is equivalent to the traditional one. Of special interest is the property that the signals generated by non-Gaussian linear SDEs tend to be sparse by construction; they also admit a concise representation in some adapted wavelet basis. Moreover, these processes can be (approximately) decoupled by applying a discrete version of the whitening operator (e.g., a finite-difference operator). The corresponding log-likelihood functions, which are nonquadratic, can be specified analytically. In particular, this allows us to uncover a Lévy processes that results in a maximum a posteriori (MAP) estimator that is equivalent to total variation. We make the connection with current methods for the recovery of sparse signals and present some examples of MAP reconstruction of MR images with sparse priors.

# References

[1] M. Unser, P. Tafti, and Q. Sun, “A unified formulation of Gaussian vs. sparse stochastic processes—Part I: Continuous-domain theory,” preprint, http://arxiv.org/abs/1108.6150.

[2] M. Unser, P.D. Tafti, “Stochastic Models for Sparse and Piecewise-Smooth Signals,” *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 989-1006, March 2011.