Sparse stochastic processes and operator-like wavelet expansions

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We introduce an extended family of continuous-domain sparse processes that are specified by a generic (non-Gaussian) innovation model or, equivalently, as solutions of linear stochastic differential equations driven by white Lvy noise. We present the functional tools for their characterization. We show that their probability distributions are infinitely divisible, which induces two distinct types of behaviorGaussian vs. sparseat the exclusion of any other. This is the key to proving that the non-Gaussian members of the family admit a sparse representation in a matched wavelet basis.

We use the characteristic form of these processes to deduce their transform-domain statistics and to precisely assess residual dependencies. These ideas are illustrated with examples of sparse processes for which operator-like wavelets outperform the classical KLT (or DCT) and result in an independent component analysis. Finally, for the case of self-similar processes, we show that the wavelet-domain probability laws are ruled by a diffusion-like equation that describes their evolution across scale.

Democracy of shearlet bases with applications to approximation and interpolation

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Shearlets are based on wavelets with composite dilation and inherit important features from wavelets. Shearlets provide (near) optimal approximation for the class of so-called cartoon-like images. Moreover, there are distribution spaces associated to them and there exist embeddings between these and classical (dyadic isotropic) inhomogeneous spaces. We prove that the shearlets are democratic bases for the shear anisotropic inhomogeneous Besov and Triebel-Lizorkin spaces (i.e. they verify the p-Temlyakov property also known as p-space property) for certain parameters. Then, we prove embeddings (or characterizations) between approximation spaces and discrete weighted Lorentz spaces (in the framework of shearlet systems) and prove (that these embeddings are equivalent to) Jackson and Bernstein type inequalities. This allows us to find (real) interpolation spaces.