TPM-T5 – Sparse stochastic processes: A unifying statistical framework for modern image processing

Instructors

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Classroom

204B

Course Motivation and Description

Sparsity and compressed sensing are very popular topics in image processing. More and more, researchers are relying on the related l1-type minimization schemes to solve a variety of ill-posed problems in imaging. The paradigm is well established with a solid mathematical foundation, although the arguments that have been put forth in the past are mostly deterministic. In this tutorial, we shall introduce the participants to the statistical side of this story. As an analogy, think of the foundational role of Gaussian stationary processes: these justify the use of the Fourier transform or DCT and lend themselves to the formulation of MMSE/MAP estimators based on the minimization of quadratic functionals.

The relevant objects here are sparse stochastic processes (SSP), which are continuous-domain processes that admit a parsimonious representation in a matched wavelet-like basis. Thus, they exhibit the kind of sparse behavior that has been exploited by researchers in recent years for designing second-generation algorithms for image compression (JPEG 2000), compressed sensing, and the solution of ill-posed inverse problems (l1 vs. l2 minimization).

The construction of SSPs is based on an innovation model that is an extension of the classical filtered-white-noise representation of a Gaussian stationary process. In a nutshell, the idea is to replace 1) the traditional white Gaussian noise by a more general continuous-domain entity (Lévy innovation) and 2) the shaping filter by a more general linear operator. We shall present the functional tools for the complete characterization of these generalized processes and the determination of their transform-domain statistics. We shall also describe self-similar models (non-Gaussian variants of fBm) that are well suited for image processing.

We shall then apply those models to the derivation of statistical algorithms for solving ill-posed problems in imaging. This allows for a reinterpretation of popular sparsity-promoting processing schemes—such as total-variation denoising, LASSO, and wavelet shrinkage—as MAP estimators for specific types of SSPs. It also suggests novel alternative Bayesian recovery procedures that minimize the estimation error (MMSE solution). The concepts will be illustrated with concrete examples of sparsity-based image processing including denoising, deconvolution, tomography, and MRI reconstruction from non-Cartesian k-space samples.

Course Outline

Introduction

- Classical reconstruction algorithms and the Gaussian hypothesis
- Variational formulations: from l2- to l1-norm minimization
- Compressed sensing

Part I: Statistical modeling
An introduction to sparse stochastic processes

- Generalized innovation model
- Statistical characterization of signals

Part II: Recovery of sparse signals
Reconstruction of biomedical images

- Discretization of inverse problems
- Generic MAP estimator (iterative reconstruction algorithm)
- Applications: deconvolution microscopy, MRI, x-ray tomography

From MAP to MMSE estimation

- MMSE estimation of Markov processes
- Iterative wavelet-domain MMSE denoising

Course Prerequisites

Basic knowledge of statistical signal processing (MAP estimation), optimization techniques (iterative algorithms), and functional analysis (Fourier transform, generalized functions, differential equations)

Distributed Material

Copies of the slides
Complete lecture notes for the tutorial (and beyond) are available on the web at http://www.sparseprocesses.org

Bibliographies

Michael UNSER is Professor and Director of EPFL's Biomedical Imaging Group, Lausanne, Switzerland. His main research area is biomedical image processing. He has a strong interest in sampling theories, multiresolution algorithms, wavelets, the use of splines for image processing, and, more recently, stochastic processes. He has published about 250 journal papers on those topics. He is the leading author of "An introduction to sparse stochastic processes", Cambridge University Press, 2014.

From 1985 to 1997, he was with the Biomedical Engineering and Instrumentation Program, National Institutes of Health, Bethesda USA, conducting research on bioimaging and heading the Image Processing Group.

Dr. Unser is a fellow of the IEEE (1999), an EURASIP fellow (2009), and a member of the Swiss Academy of Engineering Sciences. He is the recipient of several international prizes including three IEEE-SPS Best Paper Awards and two Technical Achievement Awards from the IEEE (2008 SPS and EMBS 2010).