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Restoration of echocardiograms using time warping and periodic averaging on a normalized time scale

M. UNSER[1,2], L. DONG[2], G. PELLE[2], P. BRUN[2] AND M. EDEN[1]

[1] Biomedical Engineering and Instrumentation Branch, Bldg 13, Room 3W13, National Institutes of Health, Bethesda MD 20892, USA.

[2] INSERM, Unité 138, Hôpital Henri Mondor, 94010, Créteil, France.

ABSTRACT

This paper describes an algorithm for the restoration of echocardiographic sequences of several consecutive heart beats. It is based on the estimation of the parameters of a quasi-periodic signal model. This model is fully characterized by a one-period-long reference signal and a warping function that defines the mapping between the reference and observed time-scales. Given an initial reference template, an optimal warping function is determined using dynamic programming. This function is optimal in the sense that it minimizes the mean square error between the warped template and the measured noisy signal. A reference signal estimate is then formed by averaging several cardiac cycles with reference to a normalized time-scale. The efficiency of this procedure, which may be iterated, is demonstrated quantitatively using quasi-periodic noisy test data. This method is then applied to the improvement of M-mode echocardiograms.

1. INTRODUCTION

Ultrasound echocardiography is a clinical tool that is becoming increasingly popular due to its relative low cost and its comfort and safety to the patients when compared with other medical imaging techniques. It is also particularly attractive because it is fast enough to produce real-time images of the dynamic heart, without the need for gating techniques. Its main disadvantage is the relatively poor signal quality, including a low spatial resolution and high noise levels.

To improve the signal-to-noise ratio, one may take advantage of the periodicity of the heart cycle and integrate the information from several beats as suggested by Van Ocken et al [1]. Their approach, however, is limited by the variability of the heart rate and is therefore only applicable at times in the cardiac cycle when the motion is slight [2]. In this paper, we present an approach that is based on a similar principle but that compensates for temporal variation by mapping the echo-signal onto a normalized time-scale. This is equivalent, in essence, to a non-uniform resampling of the data such that the resulting sequence is "as periodic as possible".

Our procedure uses some of the principles of a dynamic programming time-warping algorithm initially designed for single spoken word recognition [3,4]. However, it has extensions that are specific to the processing of quasi-periodic signals. The most interesting feature of this algorithm is its capability of extracting and rescaling the individual heart beats in a quasi-autonomous fashion without requiring additional information such as the position of some characteristic wave of the ECG, or the specification of the beginning of each heart beat.

2. THE QUASI-PERIODIC MODEL

The method that is described here is applicable to the enhancement of any type of echocardiographic signal, including M-mode and two-dimensional image sequences. It requires measurements to be made during several consecutive heart beats without any noticeable movement of the transducer. This last condition is essential to guarantee that the sequence of measurements are spatially registered.

For notational convenience, the signal that is acquired within a sampling interval Δt and a particular time $t_0+k\Delta t$, will be represented by an N-dimensional vector $\boldsymbol{x}_k$. Each component of this vector is a measure of the received ultrasonic energy associated with a particular spatial location along the axis of emission. A time-sequence of such measurements $\{\boldsymbol{x}_k, k=1,\ldots,K\}$ is assumed to satisfy the following model :

$$\boldsymbol{x}_k = \tilde{\boldsymbol{s}}_k + \boldsymbol{n}_k \tag{1}$$

where $\tilde{\boldsymbol{s}}_k$ is a quasi-periodic signal component and $\boldsymbol{n}_k$ is a random noise component. A sequence is said to be quasi-periodic when it can be generated by taking non-uniformly distributed samples of a signal that is initially truly periodic. The signal component of this model $\tilde{\boldsymbol{s}}_k$ is therefore fully characterized by the knowledge of a reference signal $\{\boldsymbol{s}(\tau)\}$ periodic with period T along a normalized time scale indexed by the continuous variable τ, and a sequence of non-uniform strictly increasing sampling intervals $\{\tau_k\}$. Given those quantities, the warped signal is simply defined by :

$$\tilde{\boldsymbol{s}}_k = \boldsymbol{s}(\tau=\tau_k) \tag{2}$$

In practice, $\{\boldsymbol{s}(\tau)\}$, which is assumed to be bandlimited, is characterized by L equidistant sampled values: $\{\mathrm{s}_l = \mathrm{s}(l\Delta\tau), l=1,\ldots L\}$, where $\Delta\tau=T/L$, and is evaluated from :

$$\boldsymbol{s}(\tau) = \sum_{l=1}^{L} \boldsymbol{s}_l \, \mathrm{h}(\tau-l\Delta\tau), \quad 0\leq\tau\leq T \tag{3}$$

where h(τ) is a suitable interpolation kernel. It is also convenient to define a normalized warping function which performs a mapping onto the main signal period:

$$w_k = \frac{(\tau_k - \tau_1) \bmod T}{\Delta\tau} + 1 \tag{4}$$

The relationship between the warped or quasi-periodic signal $\{\tilde{s}_k\}$ and the model parameters $\{s_l\}$ and $\{w_k\}$ is illustrated in Fig. 1 for the particular case of a one component (N=1) signal. By convention the beginning of a period (or beat) is defined as an instant where w_k=1.

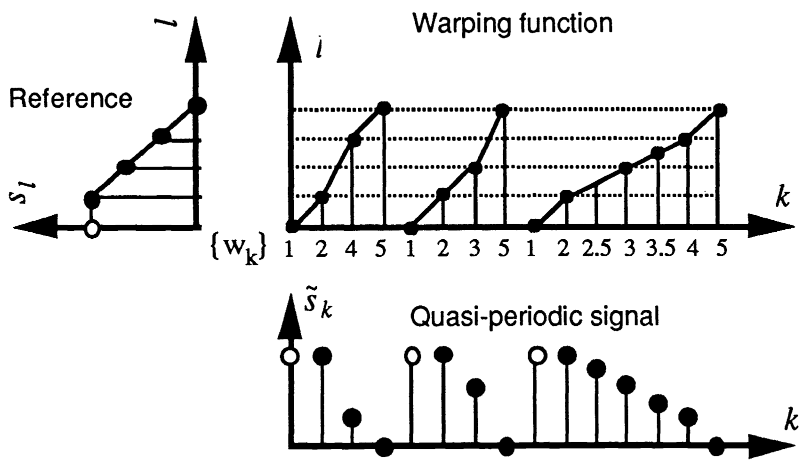

Figure 1. Generation of an univariate quasi-periodic signal.

Whenever this model is valid, the restoration of the measured noisy signal can be achieved by estimating $\{\tilde{s}_k\}$ which involves the determination of a total of (NxL + K) signal parameters : $\theta = (s_1,...,s_L; w_1, ...,w_K)$. Since N is typically large, this procedure allows a data reduction by a factor close to K/L and is expected to result in a reduction of the noise variance in the same proportion, which is approximately equal to the number of available heart beats. The most delicate part of this procedure is the estimation of the warping function $\{w_k\}$ for which an algorithm is described in the next section. In the present implementation, the estimated reference signal is determined simply by averaging on the normalized time scale once the sequence $\{w_k, k=1,..,K\}$ has been determined :

$$s_l = \frac{\sum_{k=1}^{K} \mathrm{d}(w_k - l)\, \boldsymbol{x}_k}{\sum_{k=1}^{K} \mathrm{d}(w_k - l)} \tag{5}$$

where $\mathrm{d}(x)$ is the unit sampling function : $\mathrm{d}(x)=1$ when $x=0$ and zero otherwise.

3. DETERMINATION OF THE TIME-WARPING FUNCTION

The warping function $\{w_k\}$ is determined with respect to a reference heart beat $\{\boldsymbol{r}_l, l=1,...,L\}$ by minimizing the approximation error expressed by :

$$\mathrm{d}(w_1, ..., w_K) = \sum_{k=1}^{K} ||\boldsymbol{x}_k - \tilde{\boldsymbol{r}}_k(w_1, ..., w_K)||^2 \tag{6}$$

where $\{\tilde{\boldsymbol{r}}_k, k=1,...K\}$ denotes the warped reference signal. The reference signal $\boldsymbol{r}$ is typically obtained from a segment of the original sequence $\{\boldsymbol{x}_k\}$ specified by the user so that it corresponds to one cardiac cycle. Since the specification of one cardiac period is not always very precise, we will assume that the exact duration of the heart beat is unknown and falls somewhere between $L_o=L-\Delta L$ and L, where ΔL is the only other parameter of the algorithm.

The constraints on the admissible warping functions are the following. First, $\{w_k\}$ is required to be a monotonic increasing function of k during each beat. It is expressed by the following condition :

$$\text{if } (w_k<L_o) \text{ then } (w_{k+1}>w_k) \text{ else } (w_{k+1}>w_k) \text{ or } (w_{k+1}=1) \tag{7}$$

Second, it is helpful to restrict the number of allowable warping functions by using certain continuity constraints : $\Delta_1 < w_{k+1} - w_k < \Delta_2$ (typically, $\Delta_1=1/2$ and $\Delta_2=2$). Further, in order to simplify the implementation, we choose to approximate the warping function by a succession of small linear segments the three basic types shown in Fig. 2, plus the possible transitions occurring at the end of a period. This practically means that any integral value of w_k is either followed by $(w_{k+1}=w_k+1)$ or $(w_{k+1}=w_k+2)$ or $(w_{k+1}=w_k+0.5$ and $w_{k+2}=w_k+1)$ or $(w_{k+1}=1$ if $w_k \geq L_o)$. A warping function can thus be represented by a graph joining discrete nodes in the k-l plane, as illustrated by the example in Fig. 1.

At this point we could, at least in principle, solve our optimization problem by enumerating the list of all admissible warping functions (or graphs) and calculating their corresponding approximation errors in order to retain the most favorable one. Fortunately, there is a much more efficient procedure that uses dynamic programming (DP) and takes advantage of the fact that the global criterion (6) can be decomposed into a sum of elementary contributions, each associated with an elementary transition of the warping function. This technique relies on the Bellman principle of optimality [5], which in the present case, may be re-stated as follows: If the globally most favorable warping function takes a particular value w_k at time k, then the best warping function includes, as a portion of it, the warping function that ends with the same value w_k and that is partially optimal from $k'=1,..k$. Accordingly, it is sufficient to restrict the search the partially optimal sub-trajectories. The corresponding recursive algorithm is described below.

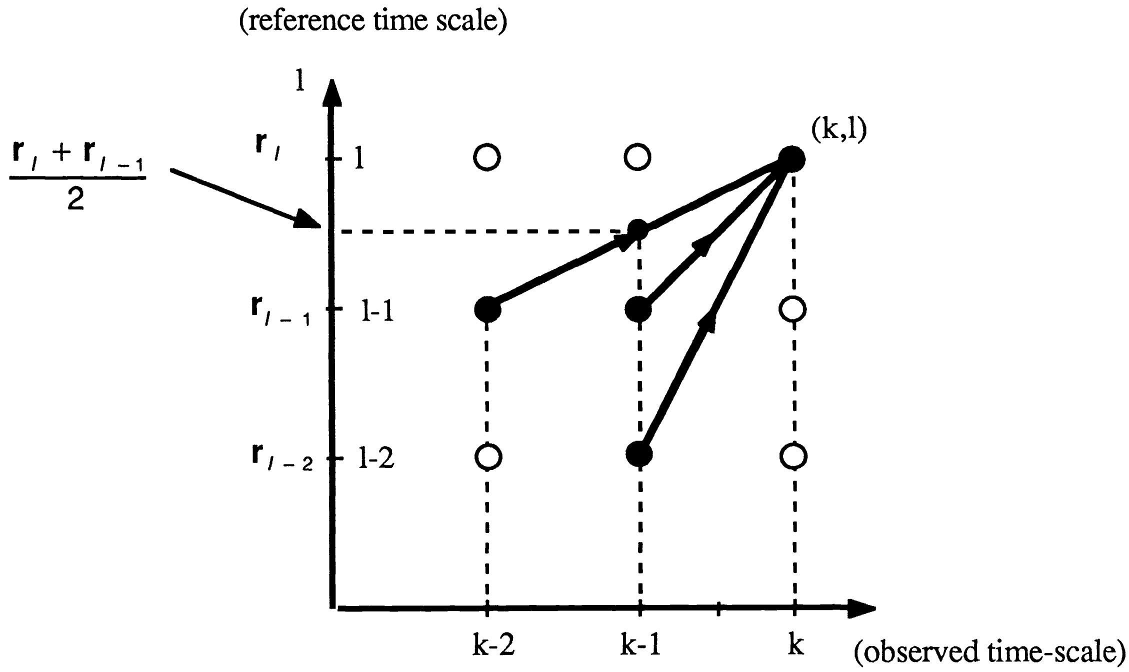

Figure 2 : Allowable transitions of the warping function (slope constraints).

Let $d_{k,l}$ denote the partially optimal cost to put into correspondence the signal segments $\{x_i, i=1,...,k\}$ and $\{r_j, j=1,...,l\}$. The algorithm iteratively scans through all values of k and l, successively. The sequence of initializations, for k=1, is the following :

$$d_{1,l} = \|x_1 - r_l\|^2 , \ (l=1,...L) \tag{8}$$

For k>1, the partially optimal distances are then updated as :

$$d_{k,1} = \|x_k - r_1\|^2 + \min\{d_{k-1,Lo}, d_{k-1,Lo+1}, ... \ d_{k-1,L}\} \tag{9}$$

$$d_{k,l} = \|x_k - r_l\|^2 + \min\{d_{k-2,l-1}, \ d_{k-1,l-1}, \ d_{k-2,l-1} + \|x_k - \frac{r_{l-1}+r_l}{2}\|^2 \}, \ (l=2,..,L) \tag{10}$$

which amounts to determining the optimal choice of the several possibilities depicted in Fig. 2. At the end of the cycle, the optimal cost function is found by searching for the minimum of $\{d_{k,l}, k=K$ and $l=1,...,L\}$. The retrieval of the optimal warping function is achieved by backtracking. This procedure is simplified by storing the most favorable predecessor of any grid point (k,l) in an auxiliary bi-dimensional array at each step of the algorithm. Since, for a given k only the values of the cost function at k-1 and k-2 are required, it is sufficient to store $\{d_{k-2,l}, l=1,..,L\}$ and $\{d_{k-1,l}, l=1,..,L\}$ in two temporary one-dimensional arrays that are updated at each increment of k and to disregard all other calculated distances.

The computational cost of the algorithm is of the order of $O(K \times L \times N)$ and is thus directly proportional to the length of the sequence (K) as well as to the number of discrete measurements (N) obtained at a given time interval. The amount of computation can be reduced by considering only a portion of the data in areas with large spatial displacements or by using a reduced number of features associated to each time frame.

4. EXPERIMENTAL RESULTS

To evaluate the performance of this algorithm, we first consider a test case generated according to the model described by Eq.(1) to (4).The parameters of the model as well as the resulting quasi-periodic signal and noisy components are displayed in Fig. 3. In this representation, the horizontal index corresponds to the time dimension of the data and each column represents an instance of the measurement vectors; each vector component is a gray level value between 0 (black) and 255 (white). The reference signal (L=32, M=64) is shown rotated by 90° in Fig. 3a. It was generated by taking a one-dimensional gaussian pulse (σ_w=7) and displacing it as a function of time according to a sinusoid with a period L=32. The warping function was chosen to provide a linear increase of the instantaneous frequency of this signal and defines the mapping between the reference (L=32) and observed (K=256) time-scales, as shown in Fig. 3b. The noisy signal (d) was obtained by adding gaussian white noise with a standard deviation of 20, resulting in a global quadratic signal-to-noise ratio (SNR) of 3.33. In a first experiment, we used the ideal reference signal displayed in Fig. 3a to estimate the warping function shown in Fig. 4b, which is in very good agreement with the initial model parameter. The signal average computed on this normalized time scale and the corresponding restored quasi-periodic signal are shown in Figs 4c and 4d, respectively. The measured SNR on the restored image is 27.2 indicating that the noise variance has been reduced by a factor that is approximately equal to (K/L). For comparison, we used the same technique to estimate the model parameters from a noise free signal (3c) which resulted in a SNR of 58. This shows that even in this idealized case, the approximation of the data is not perfect, which can be explained as follows: First, the estimated warping function is represented by short piecewise linear segments of the three types shown in Fig. 2 and will therefore only approximate the true warping function. Second, we use linear interpolation based on the two closed samples to perform the mapping from one time-scale onto another. This process can result in a slight loss of information and implies that even in a noise free case the inverse formula given by Eq. (5) is only approximate. However, for noisy measurements, the magnitude of this systematic error is usually small compared to the residual noise variance.

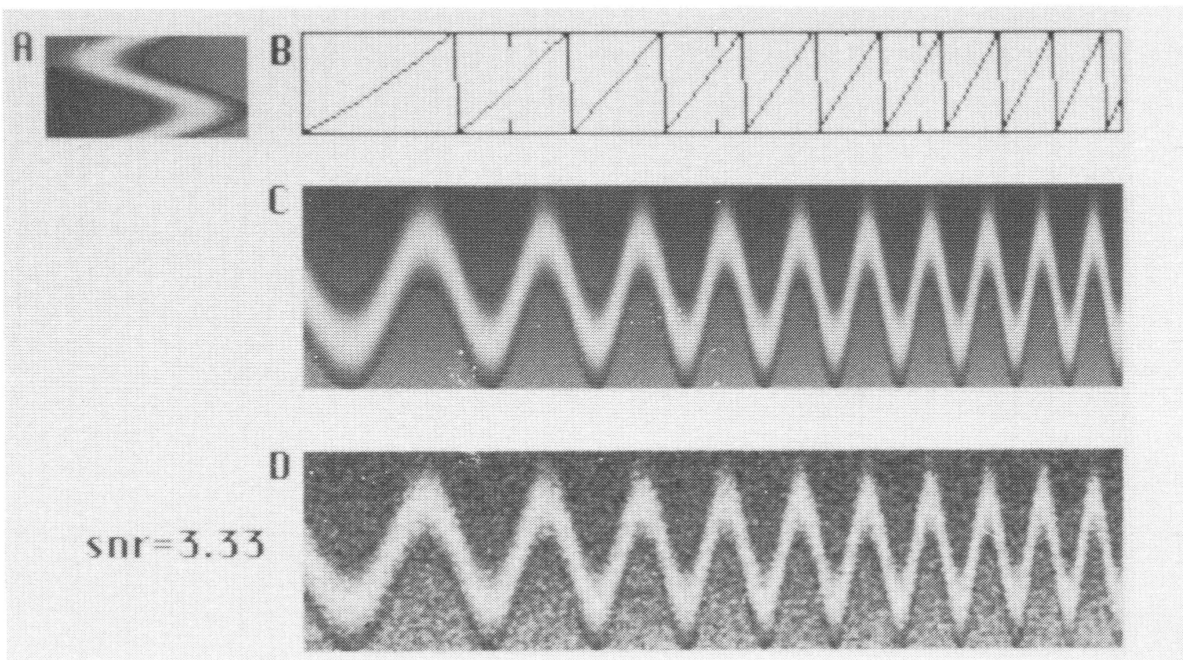

Figure 3 : Quasi-periodic test signal : (a) main period (rotated by 90°) defined on reference time-scale (L=32), (b) warping function, (c) quasi-periodic signal (K=256), (d) test signal with additive gaussian white noise.

Next, we extracted the noisy signal segment at the position marked by a small horizontal line in Fig.4b and used this as a reference template to estimate the warping function. The corresponding signal parameters obtained by our algorithm are shown in Figs. 4d and 4e, respectively. The SNR of the restored quasi-periodic image (4f) is now 21, which is still a substantial improvement over the initial noisy data. The slight decrease in performance is due to the use of a less than ideal (warped and noisy) reference to determine the warping function. Similar results (19≤SNR≤21.5) were obtained by using other signal segments to define our reference time-scale. We also found that the quality of the restoration could be slightly improved through the use of iteration (for the present case, SNR=22.34, 22.40, after 2 and 3 iterations) and that the algorithm would always converge after a relatively small number of iterations. These results suggest that the estimation procedure is efficient. The method is also quite robust since the quality of the restoration is not overly dependent on the initial choice of the reference template.

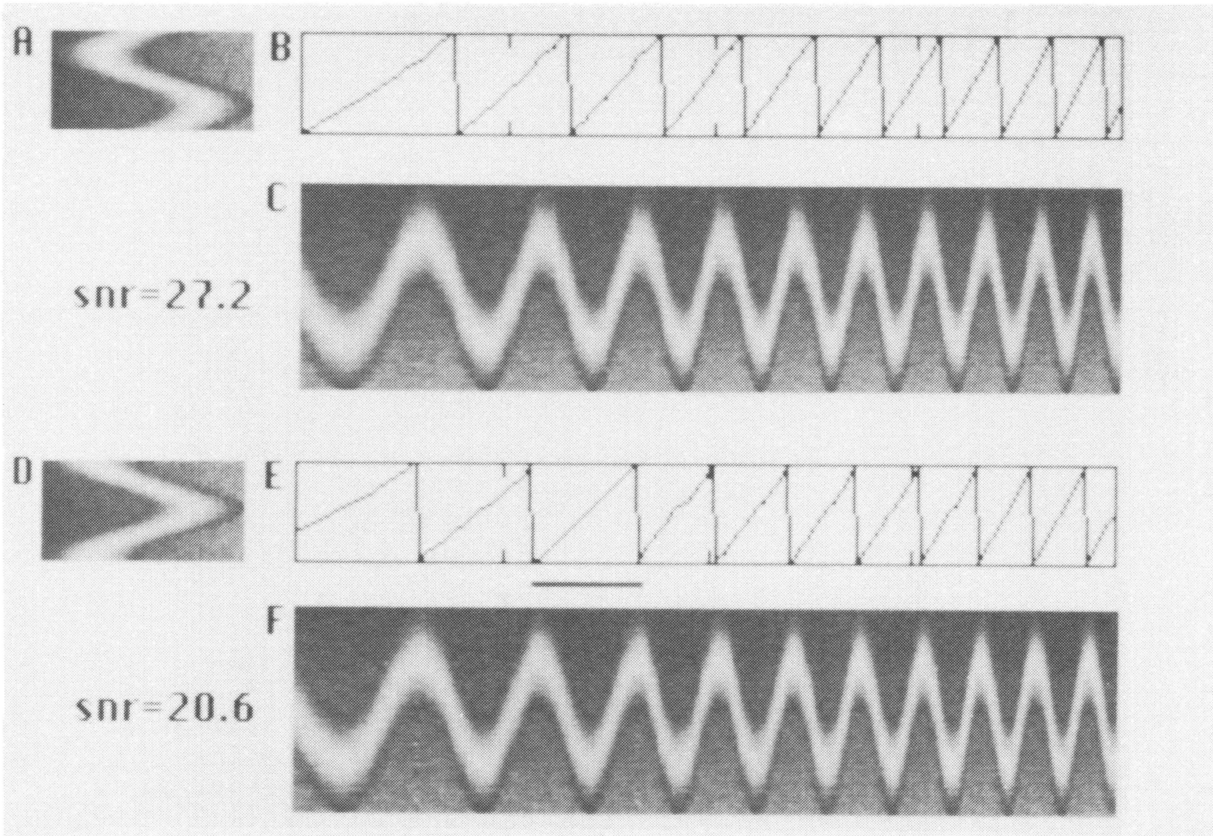

Figure 4 : Restoration of a quasi-periodic test signal. (a) estimated main period signal (rotated by 90°) (L=32) based on warping function (b); (b) warping function estimated using noise free reference template, (c) restored quasi-periodic signal (K=256) generated using parameters in (a) and (b), (d) estimated main period signal (rotated by 90°) (L=32) based on warping function (e), (e) warping function estimated using noisy reference template; (f) restored quasi-periodic signal (K=256) generated using parameters in (d) and (e).

We now consider the application of this method to real data and present some preliminary results obtained for M-mode echocardiograms recorded in the short axis direction of the left ventricle. These measurements were performed using a mechanical sector scanner (Vingmed CFM700) on normal patients in left lateral decubitus position. The spatial and temporal sampling steps were $\Delta x=1/24$ cm and $\Delta t=1/75$s, respectively. The M-mode echo signals were recorded on video tape and later digitized with 8 bits per pixel. The disadvantage of this method is that the number of consecutive time samples that can be acquired is less than 256 and is not really sufficient to take fully advantage of our algorithm. A portion of such a recording focusing on the interventricular septum and the posterior wall of the left ventricle is shown in Fig. 5a. The extracted model parameters obtained by using the reference template marked by a vertical line in Fig 5a are displayed in Fig. 5b and 5c. The restored signal (Fig. 5d) appears to be less noisy and has no apparent loss of spatial and temporal resolution. The boundaries of the interventricular septum and posterior wall are better defined and have fewer artifacts. The examination of the estimated warping function shows that all individual heart beats have been correctly detected. It is also apparent that the heart rate is highly variable which justifies the use of a time-scale renormalization procedure. These results suggest, at least qualitatively, that our method has good potential for improving the quality of echocardiograms.

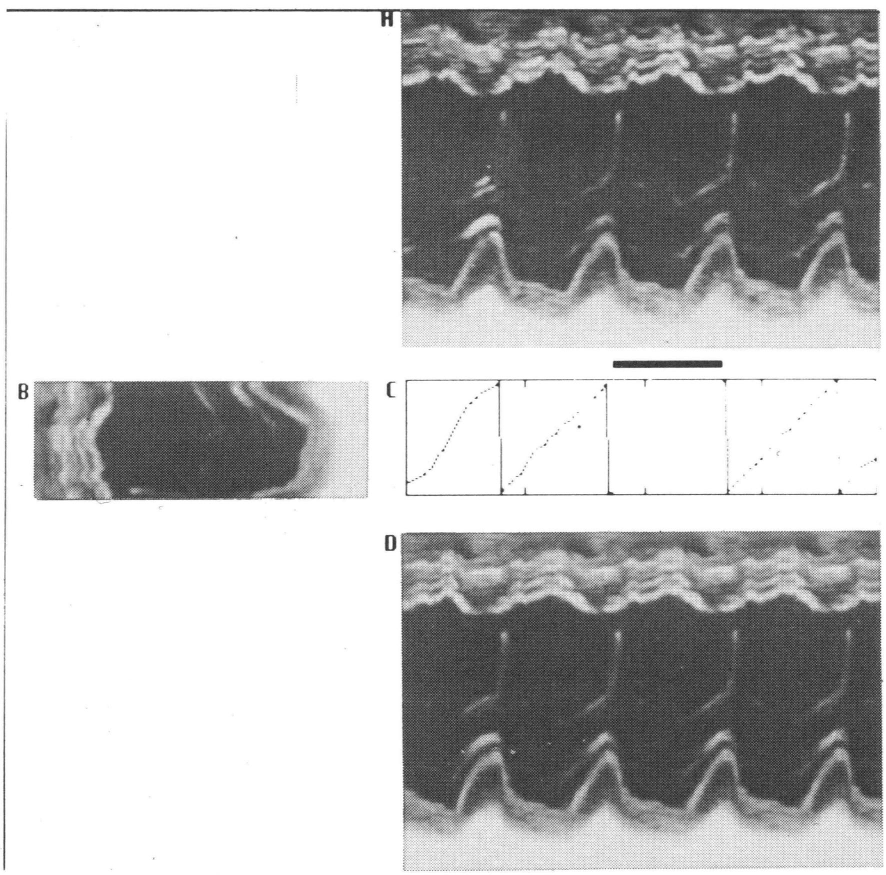

Figure 5 : Processing of an M-mode echocardiogram. (a) Noisy digitized 256×180 signal, (b) estimated main period signal (rotated by 90°) (L=65) based on warping function (c); (c) warping function estimated using noisy reference template; (d) restored quasi-periodic signal (K=256) generated using parameters in (d) and (e).

Although these preliminary results are very promising, there is obviously a need for further experimentation, particularly with longer time sequences obtained at a finer sampling rate. We are also considering schemes to compensate for small translational movements of the transducer within the axis or plane of ultrasonic emission.

5. CONCLUSION

We have presented a method that compensates for the variability of the heart rate and achieves signal restoration by averaging several cardiac cycles on a normalized time-scale. The only external requirement of this algorithm is the specification of a reference heart beat which is then used to define a reference coordinate system. A mapping of the observed signal onto this reference heart beat is evaluated by using a dynamic programming time-warping algorithm which has been extended to precisely locate the beginning and end of all heart beats. This estimation is performed by minimizing the mean-square difference between a pseudo-periodic warped signal and the actual measured data. The warping function and the average formed on the normalized time-scale are the parameters of a quasi-periodic signal model that is then used to compute a restored version of the signal.

The algorithm has been applied to a number of test signals and was found to perform well. The preliminary results that have been obtained with M-mode echocardiograms are quite encouraging. This procedure appears to be an interesting alternative to other more conventional noise reduction techniques based on spatial or temporal filtering and should provide a useful tool for improving the quality of one- or two-dimensional sequences of echocardiograms.

7. REFERENCES

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