Sparse stochastic processes:

A statistical framework for modern signal processing

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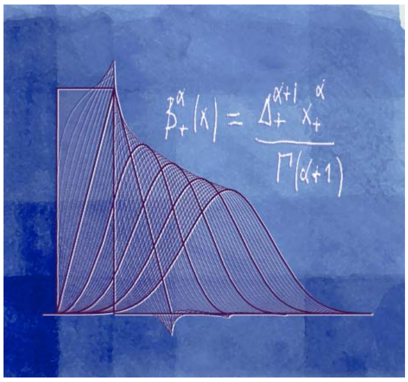


Plenary talk, Int. Conf. Syst. Sig. Im. Proc. (IWSSIP), Bucharest, July 7-9, 2013

20th century statistical signal processing

Hypothesis: Signal = stationary **Gaussian** process

Karhunen-Loève transform (KLT) is optimal for compression

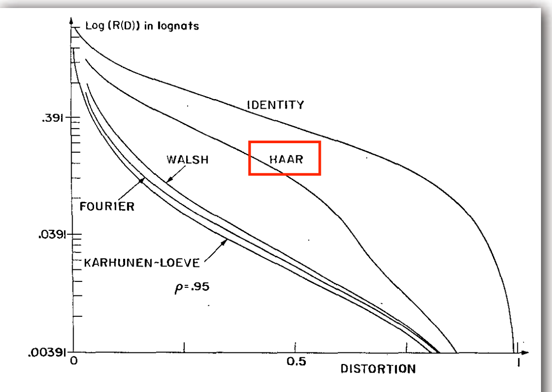


Fig. 5. Rate versus distortion for various transforms for a second-order Gauss-Markov process (ρ = 0.95, N = 256).

(Pearl et al., *IEEE Trans. Com* 1972)

DCT asymptotically equivalent to KLT

(Ahmed-Rao, 1975; U., 1984)

20th century statistical signal processing (cont'd)

Hypothesis: Signal = **Gaussian** process

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

Noise: i.i.d. Gaussian with variance σ^2

Signal covariance: $\mathbf{C}_s = \mathbb{E}\{\mathbf{s} \cdot \mathbf{s}^T\}$

Wiener filter is **optimal** for restoration/denoising

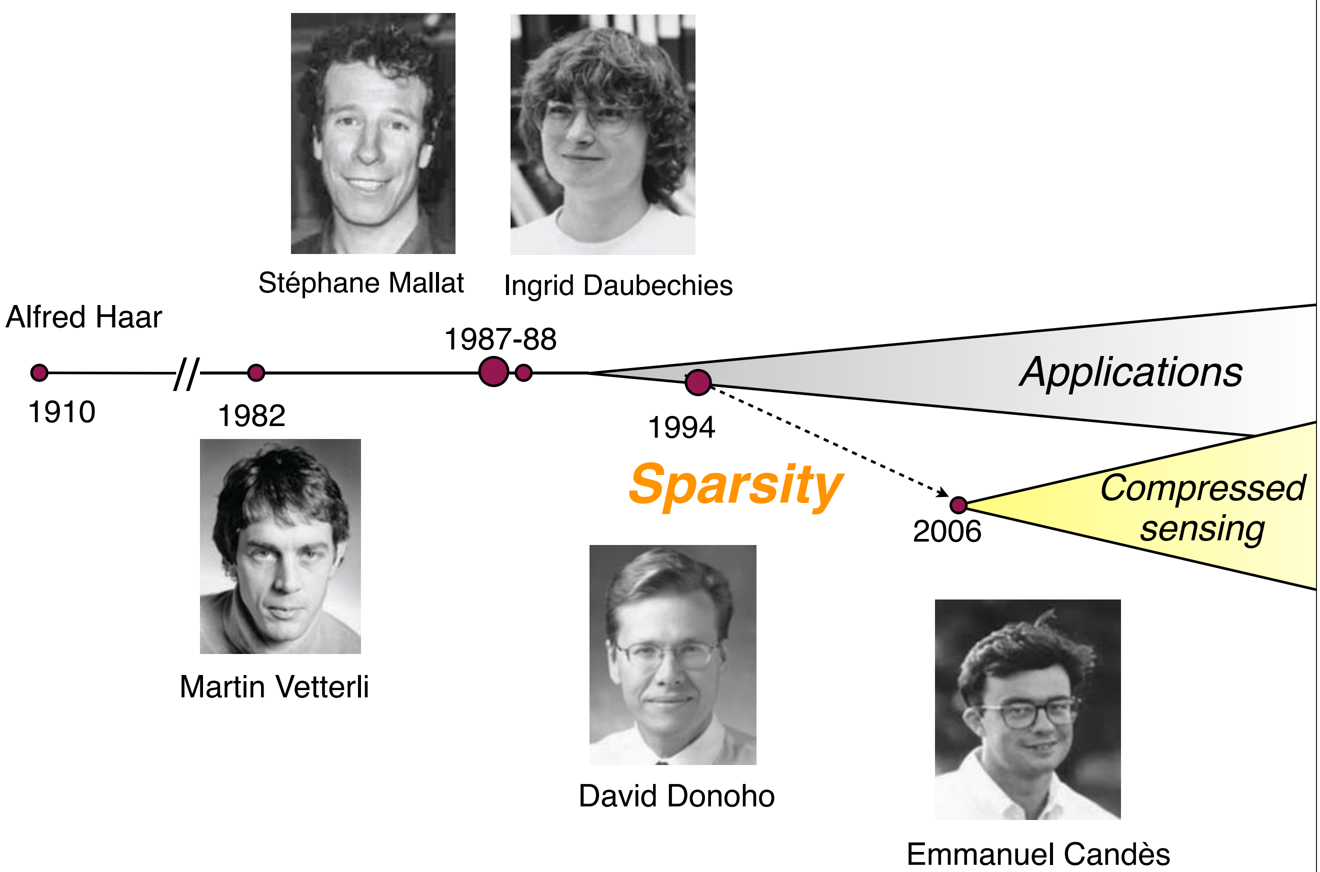
$$\mathbf{s}_{\mathrm{LMMSE}} = \mathbf{C}_s \mathbf{H}^T \left(\mathbf{H}\mathbf{C}_s\mathbf{H}^T + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{y} = \mathbf{F}_{\mathrm{Wiener}}\, \mathbf{y}$$

Wiener (LMMSE) solution = Gauss MMSE = Gauss MAP

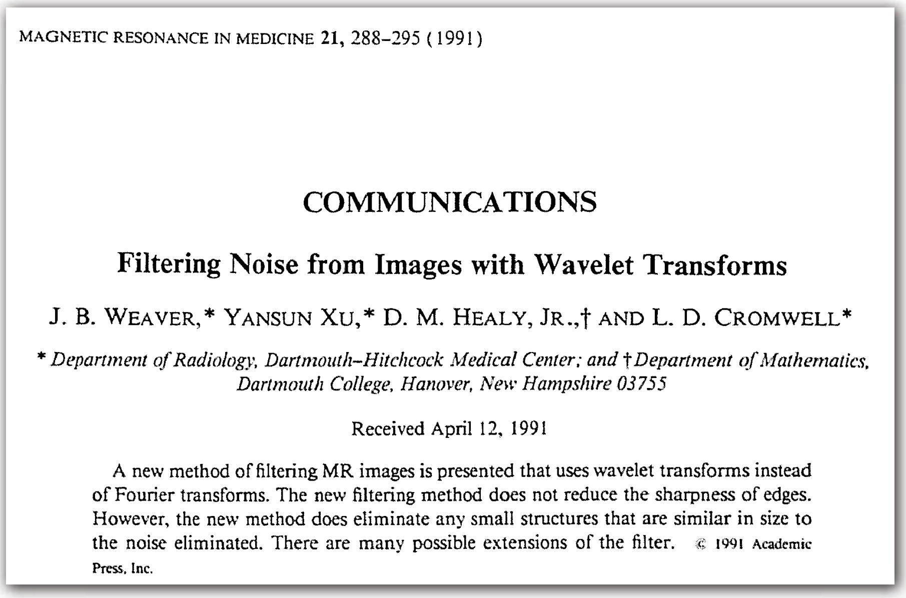
$$\mathbf{s}_{\mathrm{MAP}} = \arg\min_{\mathbf{s}} \underbrace{\frac{1}{\sigma^2}\|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2}_{\text{Data Log likelihood}} + \underbrace{\|\mathbf{C}_s^{-1/2}\mathbf{s}\|_2^2}_{\text{Gaussian prior likelihood}}$$

$\Leftrightarrow$ quadratic regularization (Tikhonov)

Then came wavelets ... and sparsity



Fact 1: Wavelets can outperform Wiener filter



MAGNETIC RESONANCE IN MEDICINE 21, 288–295 (1991)

COMMUNICATIONS

Filtering Noise from Images with Wavelet Transforms

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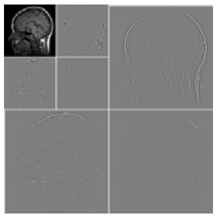
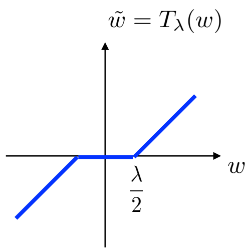
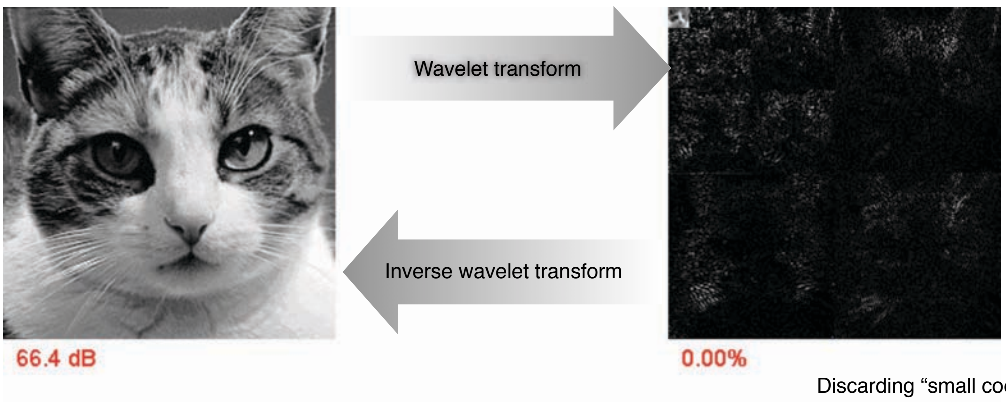
A new method of filtering MR images is presented that uses wavelet transforms instead of Fourier transforms. The new filtering method does not reduce the sharpness of edges. However, the new method does eliminate any small structures that are similar in size to the noise eliminated. There are many possible extensions of the filter.





Fact 2: Wavelet coding can outperform jpeg

$$f(\boldsymbol{x}) = \sum_{i,\boldsymbol{k}} \psi_{i,\boldsymbol{k}}(\boldsymbol{x})\, w_{i,\boldsymbol{k}}$$



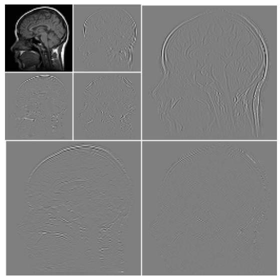
Discarding “small coefficients”

(Shapiro, *IEEE-IP* 1993)

the jpeg 2000 alliance

Fact 3: l_1 schemes can outperform l_2 optimization

$$\mathbf{s}^\star = \arg\min \underbrace{\|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2}_{\text{data consistency}} + \underbrace{\lambda \mathcal{R}(\mathbf{s})}_{\text{regularization}}$$



- Wavelet-domain regularization

  Wavelet expansion: $\mathbf{s} = \mathbf{W}\mathbf{v}$ (typically, sparse)

  Wavelet-domain sparsity-constraint: $\mathcal{R}(\mathbf{s}) = \|\mathbf{v}\|_{\ell_1}$ with $\mathbf{v} = \mathbf{W}^{-1}\mathbf{s}$

  Iterated shrinkage-thresholding algorithm (ISTA, FISTA)

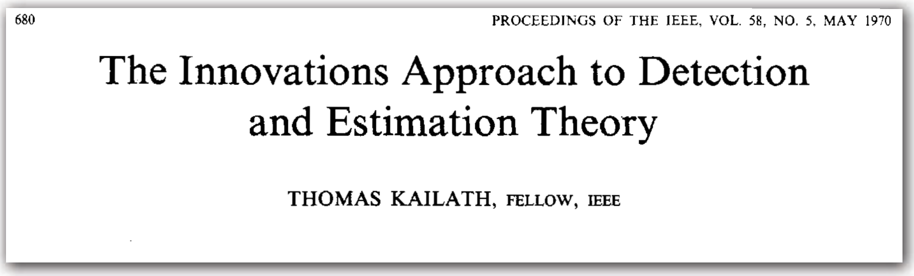
  (Nowak et al., Daubechies et al. 2004)

- ℓ_1 regularization (Total variation)

  $\mathcal{R}(\mathbf{s}) = \|\mathbf{L}\mathbf{s}\|_{\ell_1}$ with $\mathbf{L}$: gradient (Rudin-Osher, 1992)

  Iterative reweighted least squares (IRLS) or FISTA

Quest for a unifying framework: the precursor


680 PROCEEDINGS OF THE IEEE, VOL. 58, NO. 5, MAY 1970

The Innovations Approach to Detection and Estimation Theory

THOMAS KAILATH, FELLOW, IEEE

- Gaussian stationary processes as a filtered white noise

  $s(t) = (h * w)(t)$ $\Phi_s(\boldsymbol{\omega}) = |H(\boldsymbol{\omega})|^2 \, \Phi_w(\boldsymbol{\omega}) \propto |H(\boldsymbol{\omega})|^2$

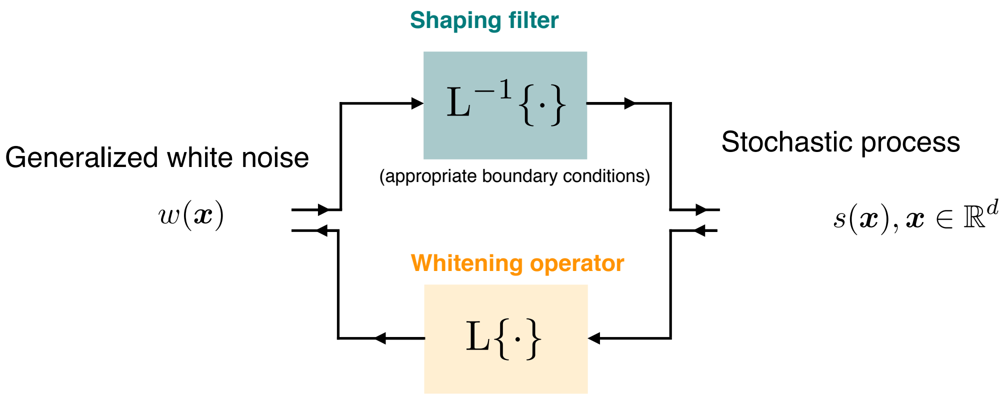
  Frequency response (shaping filter): $H(\omega) = \int_{\mathbb{R}} h(t) e^{-j\omega t} \mathrm{d}t$

- Whitening operator

  Innovation: $\mathrm{L}s(t) = w(t)$ $L(\omega) = \frac{1}{H(\omega)}$

  w is **Gaussian** stationary and independent at every point

Continuous-domain innovation model



Our work: the **non-gaussian** part of this story

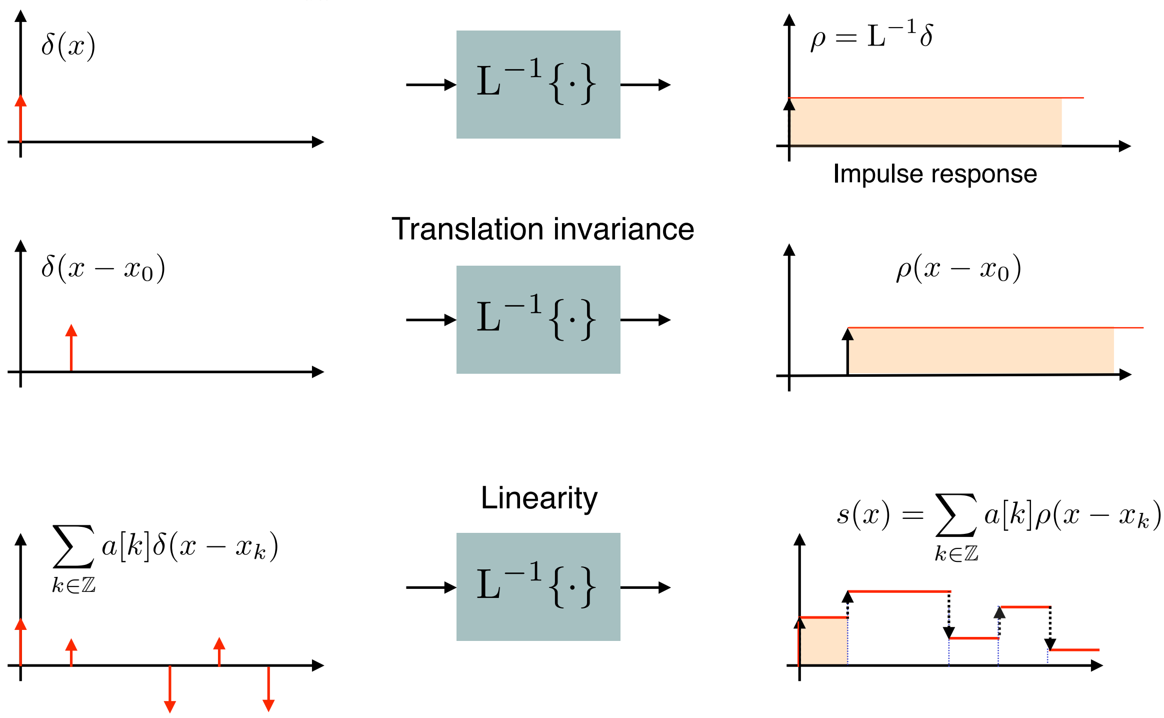
Main finding: it is **necessarily** sparse (***infinitely divisible***)

Why? ... as explained by our current research ...

(invoking powerful theorems in functional analysis:
Bochner-Minlos, Gelfand, Schoenberg & Lévy-Khinchine)

Innovation-based synthesis of splines

$\mathrm{L} = \frac{\mathrm{d}}{\mathrm{d}x} = \mathrm{D} \quad \Rightarrow \quad \mathrm{L}^{-1}$: integrator



OUTLINE

- Sparse stochastic processes
  - Generalized innovation model
  - Gelfand's theory of generalized stochastic processes
  - Statistical characterization of sparse stochastic processes
  - Lévy processes and their generalization
  - Fractal processes: Gaussian vs. sparse
- Applications
  - Modeling of signals (audio)
  - Algorithms for sparse signal recovery as MAP estimators
  - Optimal denoising (MMSE)
  - Sparse representations, optimal transforms

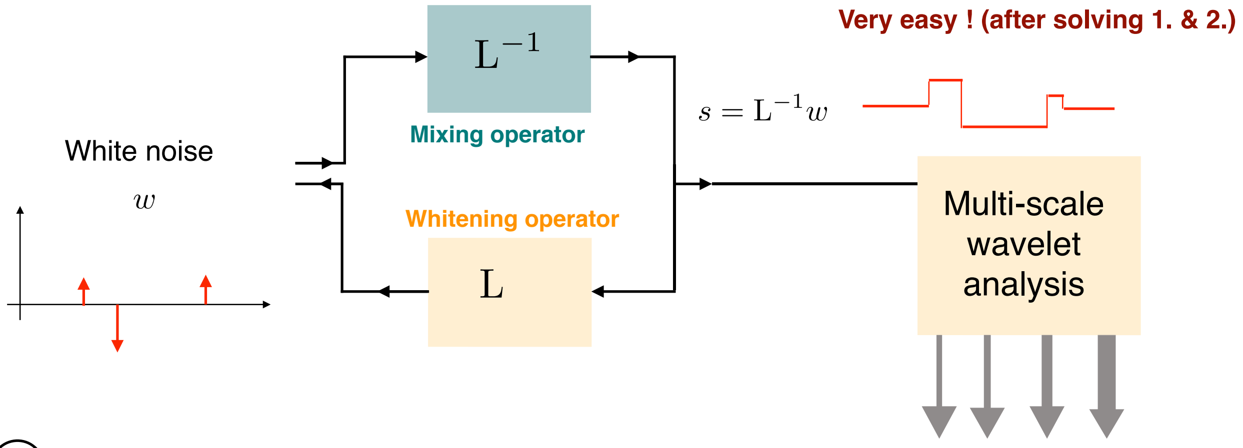
Road map for theory of sparse processes

(2) Specification of inverse operator

Functional analysis solution of SDE

(3) Characterization of generalized stochastic process

Very easy ! (after solving 1. & 2.)



(1) Characterization of continuous-domain white noise

Higher mathematics: generalized functions (Schwartz)
measures on topological vector spaces

Gelfand's theory of *generalized stochastic processes*
Infinite divisibility (Lévy-Khintchine formula)

(4) Characterization of transform-domain statistics

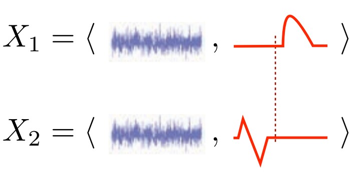
Easy when: $\psi_i = \mathrm{L}^*\phi_i$

Generalized innovation process

- Difficulty 1: $w \neq w(\boldsymbol{x})$ is too rough to have a pointwise interpretation
- Difficulty 2: w is an infinite-dimensional random entity; its "pdf" can be formally specified by a measure $\mathscr{P}_w(E)$ where $E \subseteq \mathcal{S}'(\mathbb{R}^d)$
- Axiomatic definition (Gelfand-Vilenkin 1964)

w is a generalized innovation process (or continuous-domain white noise) over $\mathcal{S}'(\mathbb{R}^d)$ if

1. ***Observability*** : $X = \langle w, \varphi \rangle$ is a well-defined random variable for any test function $\varphi \in \mathcal{S}(\mathbb{R}^d)$.
2. ***Stationarity*** : $X_{\boldsymbol{x}_0} = \langle w, \varphi(\cdot - \boldsymbol{x}_0) \rangle$ is identically distributed for all $\boldsymbol{x}_0 \in \mathbb{R}^d$.
3. ***Independent atoms*** : $X_1 = \langle w, \varphi_1 \rangle$ and $X_2 = \langle w, \varphi_2 \rangle$ are independent whenever φ_1 and φ_2 have non-intersecting support.



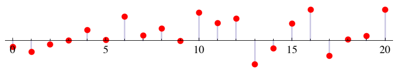
- Characteristic functional $(\omega \rightarrow \varphi)$

$$\widehat{\mathscr{P}}_w(\varphi) = \mathbb{E}\{e^{j\langle w, \varphi \rangle}\} = \int_{\mathcal{S}'} e^{j\langle s, \varphi \rangle} \mathscr{P}_w(ds)$$

Defining Gaussian noise: discrete vs. continuous

Lévy exponent: $\log \hat{p}_X(\omega) = f(\omega) = -\frac{1}{2}\omega^2$

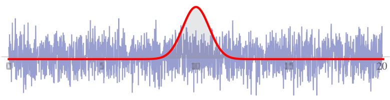
- Discrete white Gaussian noise



$X = (X_1, \dots, X_N)$ with X_n i.i.d standardized Gaussian

Characteristic function: $\hat{p}_X(\boldsymbol{\omega}) = g(\boldsymbol{\omega}) = \exp\left(\sum_{n=1}^{N} f(\omega_n)\right) = e^{-\frac{1}{2}\|\boldsymbol{\omega}\|^2}$

- Continuous-domain white Gaussian noise

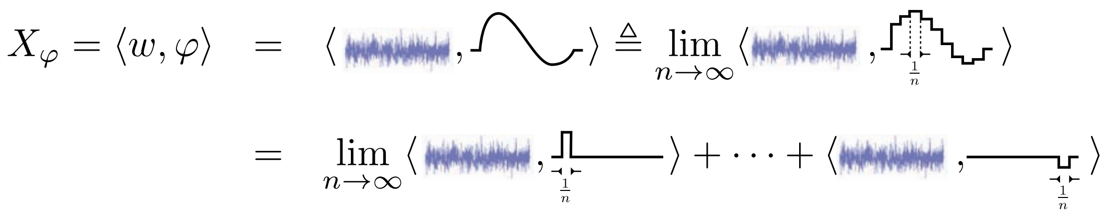


Infinite-dimensional entity w with generic observations $X_n = \langle w, \varphi_n \rangle$

Characteristic functional: $\widehat{\mathscr{P}}_w(\varphi) = G(\varphi) = e^{-\frac{1}{2}\|\varphi\|^2_{L_2}} = \exp\left(\int_{\mathbb{R}} f(\varphi(x))\mathrm{d}x\right)$

$$\hat{p}_{X_n}(\omega) = \mathbb{E}\{e^{j\omega\langle w, \varphi_n\rangle}\} = \mathbb{E}\{e^{j\langle w, \omega\varphi_n\rangle}\} = \widehat{\mathscr{P}}_w(\omega\varphi_n) = e^{-\frac{1}{2}\omega^2\|\varphi_n\|^2_{L_2}}$$

Characterization of generalized innovation



Theorem

Let w be a generalized stochastic process such that $X_{\mathrm{id}} = \langle w, \mathrm{rect}\rangle$ is well-defined. Then, w is a generalized innovation (white noise) over $\mathcal{S}'(\mathbb{R}^d)$ **if and only if** its characteristic form is given by

$$\widehat{\mathscr{P}}_w(\varphi) = \mathbb{E}\{e^{j\langle w, \varphi\rangle}\} = \exp\left(\int_{\mathbb{R}^d} f(\varphi(\boldsymbol{x}))\mathrm{d}\boldsymbol{x}\right)$$

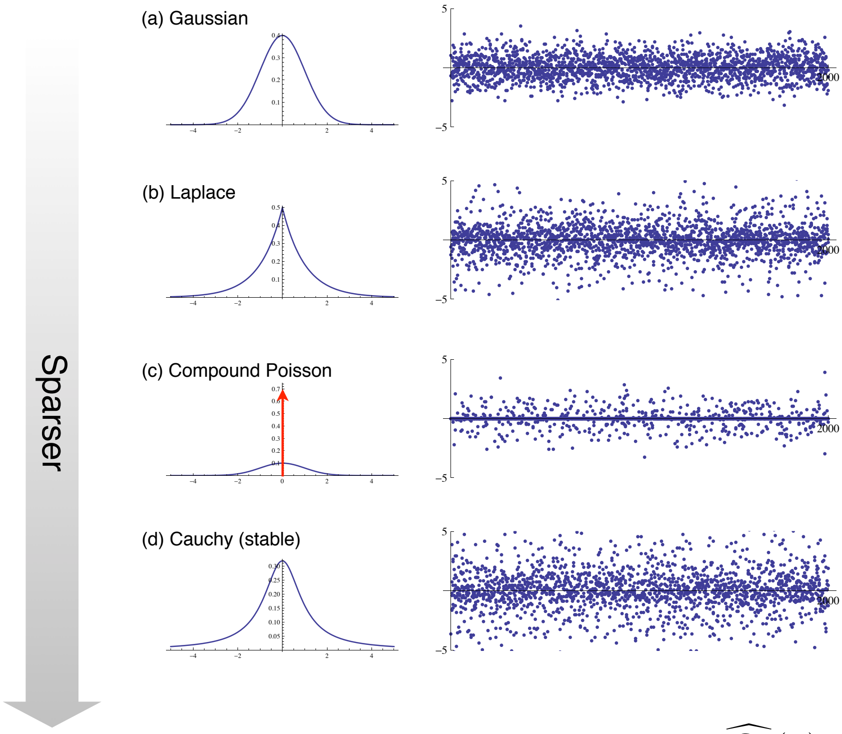
where $f(\omega)$ is a **valid Lévy exponent** (in fact, the Lévy exponent of X_{id}).
Moreover, the random variables $X_\varphi = \langle w, \varphi\rangle$ are all **infinitely divisible** with modified Lévy exponent

$$f_\varphi(\omega) = \int_{\mathbb{R}^d} f(\omega\varphi(\boldsymbol{x}))\mathrm{d}\boldsymbol{x}$$

(Gelfand-Vilenkin 1964; Amini-U. under review)

Examples of id noise distributions

$p_{\mathrm{id}}(x)$ Observations: $X_n = \langle w, \mathrm{rect}(\cdot - n)\rangle$



$f(\omega) = -\frac{1}{2\sigma_0^2}|\omega|^2$

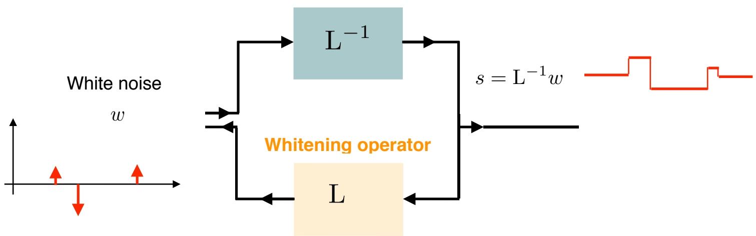
$f(\omega) = \log\left(\frac{1}{1+\omega^2}\right)$

$f(\omega) = \lambda \int_{\mathbb{R}} (e^{jx\omega} - 1) p(x) \mathrm{d}x$

$f(\omega) = -s|\omega|$

Complete mathematical characterization: $\widehat{\mathscr{P}}_w(\varphi) = \exp\left(\int_{\mathbb{R}^d} f(\varphi(\boldsymbol{x}))\mathrm{d}\boldsymbol{x}\right)$

Steps 2 + 3: Characterization of sparse process



- Abstract formulation of innovation model

$$s = \mathrm{L}^{-1}w \quad \Leftrightarrow \quad \forall \varphi \in \mathcal{S}, \quad \langle \varphi, s\rangle = \langle \varphi, \mathrm{L}^{-1}w\rangle = \langle \underbrace{\mathrm{L}^{-1*}\varphi}, w\rangle$$

$$\Rightarrow \quad \widehat{\mathscr{P}}_s(\varphi) = \mathbb{E}\{e^{j\langle s,\varphi\rangle}\} = \widehat{\mathscr{P}}_w(\mathrm{L}^{-1*}\varphi) = \exp\left(\int_{\mathbb{R}^d} f\left(\mathrm{L}^{-1*}\varphi(\boldsymbol{x})\right)\mathrm{d}\boldsymbol{x}\right)$$

Sufficient condition for existence:
L^{-1*} continuous operator: $\mathcal{S}(\mathbb{R}^d) \to L_p(\mathbb{R}^d)$

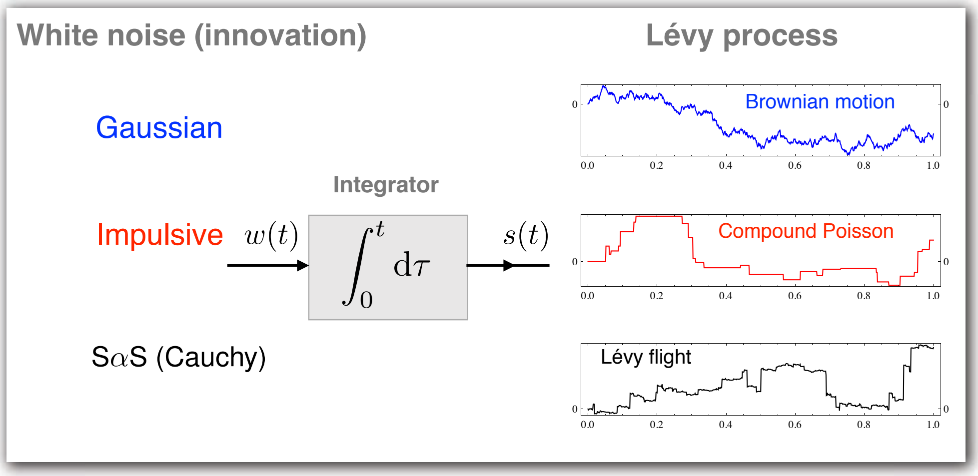
(U.-Tafti-Sun, preprint ArXiv 2011)

Example 1: Lévy processes

$\mathrm{D}s = w$ (unstable SDE !)

$$s = \mathrm{D}_0^{-1} w \quad \Leftrightarrow \quad \forall \varphi \in \mathcal{S}, \quad \langle \varphi, s \rangle = \langle \mathrm{D}_0^{-1*} \varphi, w \rangle$$

$$\mathrm{D}_0^{-1} \varphi(t) = \int_0^t \varphi(\tau) \mathrm{d}\tau$$

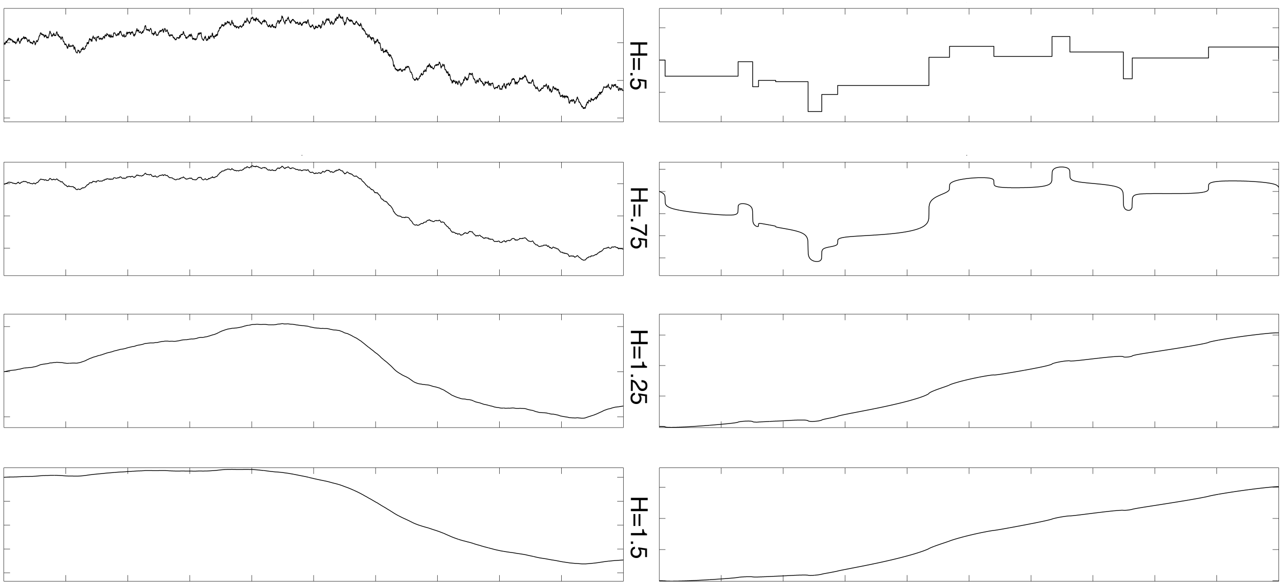


(Wiener 1923)

(Paul Lévy circa 1930)

Example 2: Self-similar processes

$$\mathrm{L} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad (j\omega)^{H+\frac{1}{2}} \quad \Rightarrow \quad \mathrm{L}^{-1}\text{: fractional integrator}$$



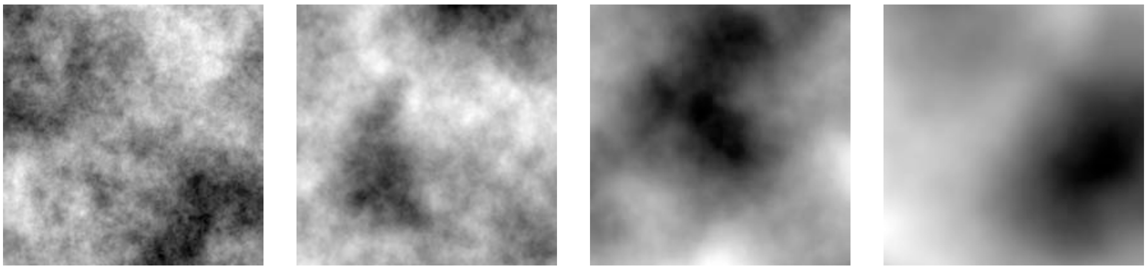
Gaussian | **Sparse (generalized Poisson)**

Fractional Brownian motion (Mandelbrot, 1968) | (U.-Tafti, *IEEE-SP* 2010)

Scale- and rotation-invariant processes

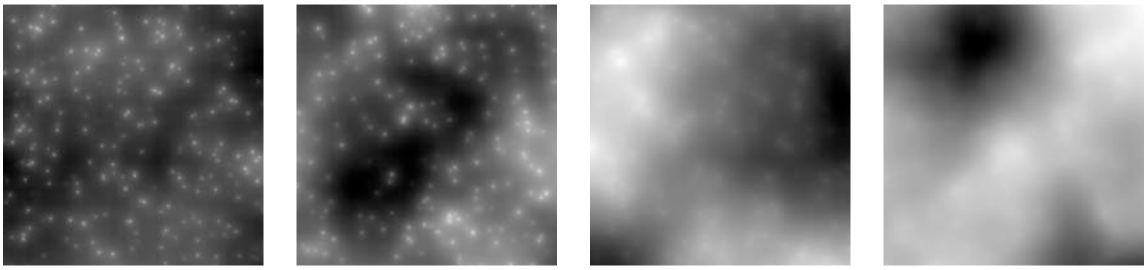
Stochastic partial differential equation : $(-\Delta)^{\frac{H+1}{2}} s(\boldsymbol{x}) = w(\boldsymbol{x})$

Gaussian



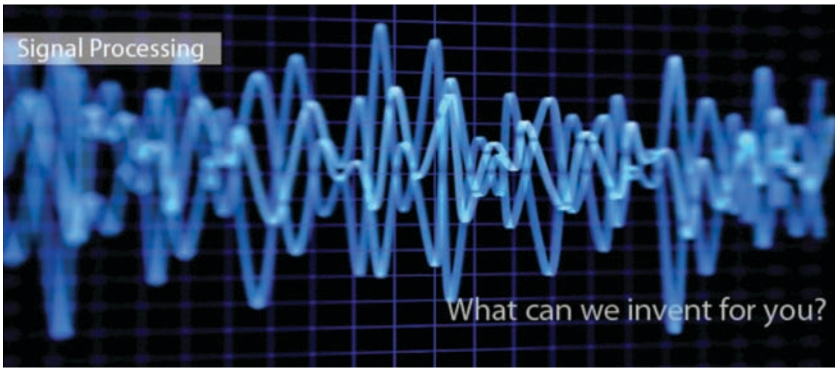
H=.5 H=.75 H=1.25 H=1.75

Sparse (generalized Poisson)



(U.-Tafti, *IEEE-SP* 2010)

A brief panorama of applications

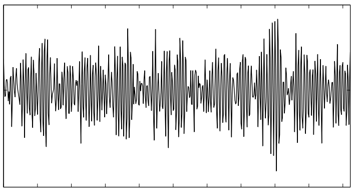


A1. Signal modeling (Audio)

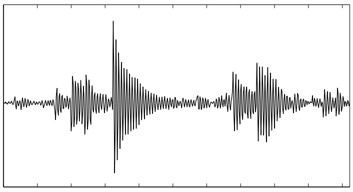
- Sparse, bandpass processes

$$\mathrm{L} = \frac{\mathrm{d}^n}{\mathrm{d}t^n} + a_{n-1}\frac{\mathrm{d}^{n-1}}{\mathrm{d}t^{n-1}} + \cdots + a_1\frac{\mathrm{d}}{\mathrm{d}t} + a_0\mathrm{I}$$

poles = [−.05 + jπ/2, −.05 − jπ/2], zeros = []



(a) Gaussian



(b) Alpha stable α=1.2

- Mixed sparse processes: $s_{\mathrm{mix}} = s_1 + \cdots + s_M$

$$\widehat{\mathscr{P}}_{s_{\mathrm{mix}}}(\varphi) = \prod_{m=1}^{M} \widehat{\mathscr{P}}_{s_m}(\varphi) = \exp\left(\int_{\mathbb{R}} \sum_{m=1}^{M} f_m(\mathrm{L}_m^{-1*}\varphi(t))\mathrm{d}t\right)$$

Gaussian (Am)

generalized Lévy (Am, SαS)

A2. Biomedical imaging: MAP reconstruction

- Innovation model of the signal

$$\mathrm{L}s = w$$
$$s = \mathrm{L}^{-1}w$$

- Signal decoupling: discrete version of operator

$u(\boldsymbol{x}) = \mathrm{L_d}\, s(\boldsymbol{x}) \quad \Leftrightarrow \quad \mathbf{u} = \mathbf{Ls}$ (matrix notation)

Generalized B-spline
$\beta_{\mathrm{L}} = \mathrm{L_d}\mathrm{L}^{-1}\delta$

- Statistical characterization
  - $X = [\mathbf{u}]_n$ identically distributed (approx. independent)
  - Probability density function: $p_X(x) = \mathcal{F}^{-1}\{\widehat{\mathscr{P}}_w(\omega\beta_{\mathrm{L}})\}(x)$
  - Potential function: $\Phi_X(x) = -\log p_X(x)$

- Maximum a posteriori (MAP) estimator for AWN

$$\mathbf{s}^\star = \mathrm{argmin}\left(\tfrac{1}{2}\|\mathbf{g} - \mathbf{Hs}\|_2^2 + \sigma^2 \textstyle\sum_n \Phi_X([\mathbf{Ls}]_n)\right)$$

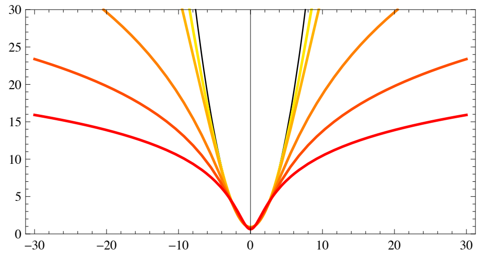
(Bostan et al., *IEEE-IP* 2013)

MAP estimator: special cases

$$\mathbf{s}^\star = \operatorname{argmin}\left(\frac{1}{2}\|\mathbf{g}-\mathbf{Hs}\|_2^2 + \sigma^2 \sum_n \Phi_X([\mathbf{Ls}]_n)\right)$$

Sparser

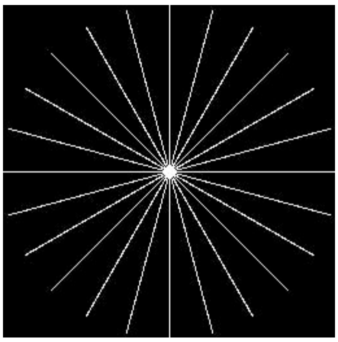
- Gaussian: $p_X(x) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-x^2/(2\sigma_0^2)}$ $\Rightarrow$ $\Phi_X(x) = \frac{1}{2\sigma_0^2} x^2$
- Laplace: $p_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}$ $\Rightarrow$ $\Phi_X(x) = \lambda|x|$
- Student: $p_X(x) = \frac{1}{B\left(r, \frac{1}{2}\right)} \left(\frac{1}{x^2+1}\right)^{r+\frac{1}{2}}$ $\Rightarrow$ $\Phi_X(x) = (r + \frac{1}{2}) \log(1+x^2)$



Student potentials: $r = 2, 4, 8, 32$ (fixed variance)

MRI: Shepp-Logan phantom

Original SL Phantom



Fourier Sampling Pattern
12 Angles

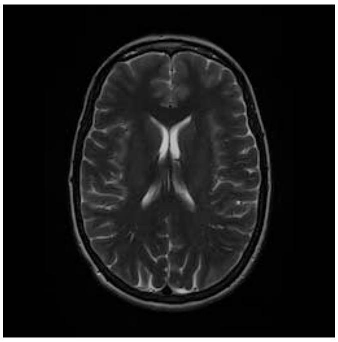
Laplace prior (TV)

Student prior (log)

L : gradient

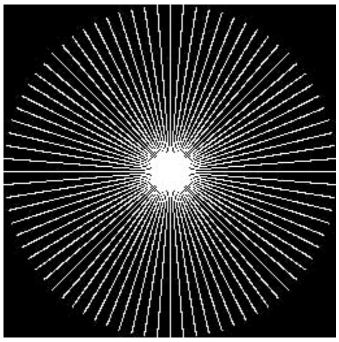
Optimized parameters

MRI reconstruction



Real T2 Brain Image

MR Angiography Image



k-space sampling pattern

40 radial lines

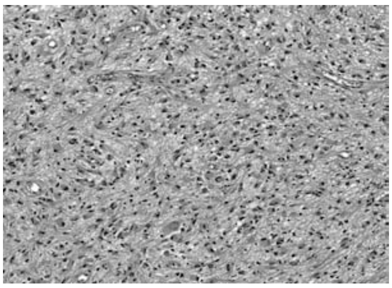
L : gradient

Optimized parameters

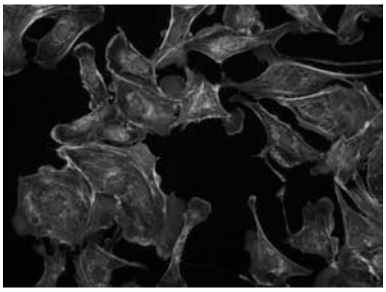
Reconstruction results in dB

	Gaussian Estimator	Laplace Estimator	Student's Estimator
T2 brain Image	8.71	**16.08**	15.79
MR Angiography Image	6.31	14.48	**14.97**

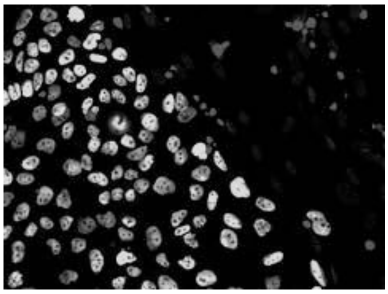
2D deconvolution experiment



Astrocytes cells



bovine pulmonary artery cells



human embryonic stem cells

Disk shaped PSF (7x7)

L : gradient

Optimized parameters

Deconvolution results in dB

	Gaussian Estimator	Laplace Estimator	Student's Estimator
Astrocytes cells	**12.18**	10.48	10.52
Pulmonary cells	16.90	**19.04**	18.34
Stem cells	15.81	20.19	**20.50**

(Bostan et al., *IEEE-IP* 2013)

A3. Optimal denoising: MMSE formulation

- Measurement model: $\mathbf{y} = \mathbf{x} + \mathbf{n}$

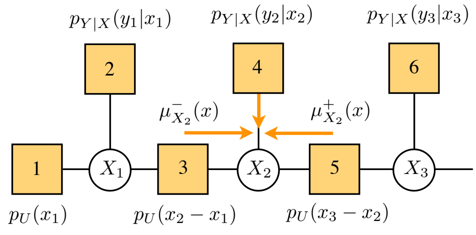
  $\mathbf{n}$: additive white Gaussian noise

  $\mathbf{x}$: sparse first-order process (e.g., Lévy flight)

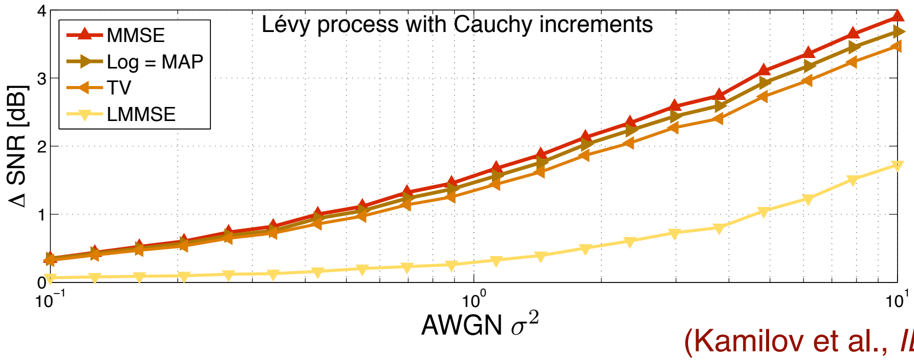
  $\mathbf{u} = \mathbf{L}\mathbf{s}$: discrete innovation (i.i.d. and infinite divisible)

Optimal estimator: $\mathbf{x}_{\text{MMSE}} = E\{\mathbf{x}|\mathbf{y}\}$

$$p(\mathbf{x}|\mathbf{y}) = \frac{1}{Z}\prod_{n=1}^{N} p_{Y|X}(y_n|x_n)\prod_{n=1}^{N} p_U\big(\underbrace{x_n - x_{n-1}}_{u_n}\big)$$



Belief propagation algorithm



(Kamilov et al., *IEEE-SP* 2013)

A4. Sparse representations, optimal transforms

- Innovation model (SDE)

$$\begin{aligned} \mathrm{L}s &= w \\ s &= \mathrm{L}^{-1}w \end{aligned}$$

- Admissible basis function: $\psi_{i,k} = \mathrm{L}^*\phi_{i,k}$ with $\phi_{i,k} \in L_p(\mathbb{R}^d)$

- Signal expansion = equivalent white-noise analysis

$$Y = \langle \psi_{i,k}, s\rangle = \langle \mathrm{L}^*\phi_{i,k}, \mathrm{L}^{-1}w\rangle = \langle \phi_{i,k}, w\rangle$$

$$\Longrightarrow \quad \hat{p}_Y(\omega) = \widehat{\mathscr{P}_w}(\omega\phi_{i,k})$$

- Statistical implications
  - Transform-domain pdfs are infinitely divisible
  - Quality of decoupling depends upon support of wavelet/smoothing kernel $\phi_{i,k}$

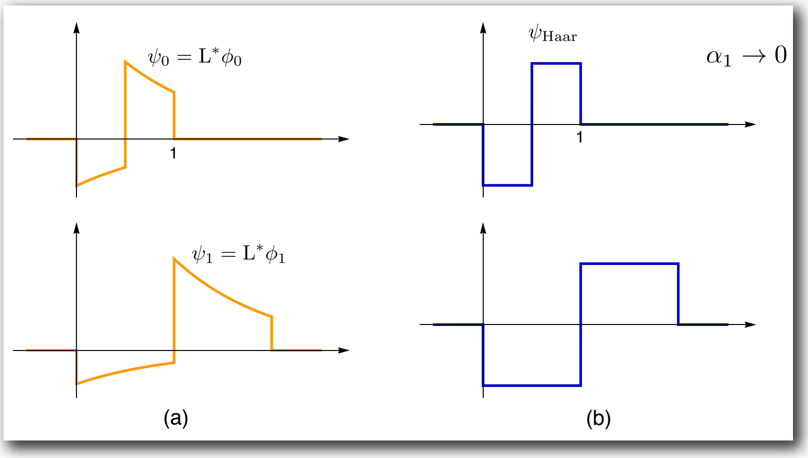
Operator-like wavelets for sparse AR(1) processes

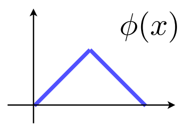
Innovation model: $\mathrm{L}s = w \quad \Leftrightarrow \quad s = \mathrm{L}^{-1}w$ with $\mathrm{L} = (\mathrm{D} - \alpha_1 \mathrm{I})$

Operator-like wavelet: $\psi_i = \mathrm{L}^* \phi_i$ with ϕ_i: smoothing kernel

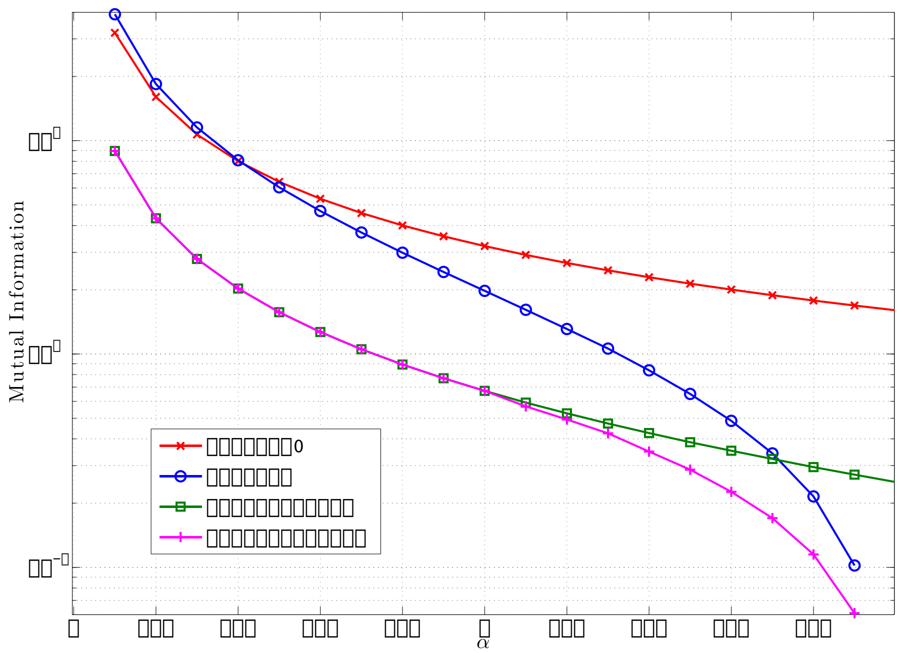
(Khalidov-U., 2006)

Wavelet analysis: $\langle s, \psi_i(\cdot - t_0) \rangle = \langle \mathrm{L}^{-1}w, \mathrm{L}^* \phi_i(\cdot - t_0) \rangle = \langle w, \phi_i(\cdot - t_0) \rangle$





Orthogonal expansion of a SαS Lévy process

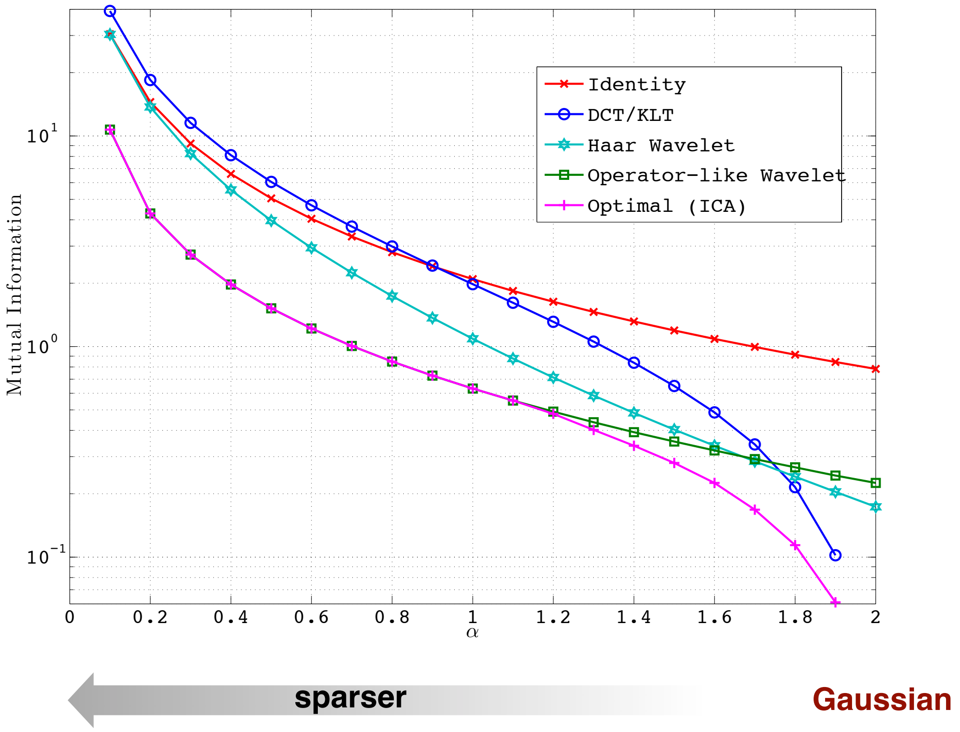


sparser

Gaussian

(Pad-U. ICASSP'13, SPARS'13)

Orthogonal expansion of a SαS AR(1) process



(Pad-U. ICASSP'13, SPARS'13)

$e^{\alpha_1} = 0.9, M = 64$

CONCLUSION

- Unifying continuous-domain innovation model
  - Backward compatibility with classical Gaussian theory
  - Operator-based formulation: Lévy-driven SDEs
  - **Gaussian** vs. **sparse** (generalized Poisson, student, SαS)
  - Focus on unstable SDEs ⇒ non-stationary, self-similar processes
- Wavelet analysis vs. regularization
  - Central role of B-spline (see papers)
  - Sparsification/decoupling via "operator-like" behavior
- Theoretical framework for sparse signal recovery
  - Analytical determination of PDF in **any** transformed domain
  - Predictive power: transform coding/denoising (**facts 1, 2, 3**)
  - New statistically-founded sparsity priors
  - Derivation of estimators (MAP vs. MMSE): link with LASSO and l_1 methods for sparse signal recovery (Compressed sensing)

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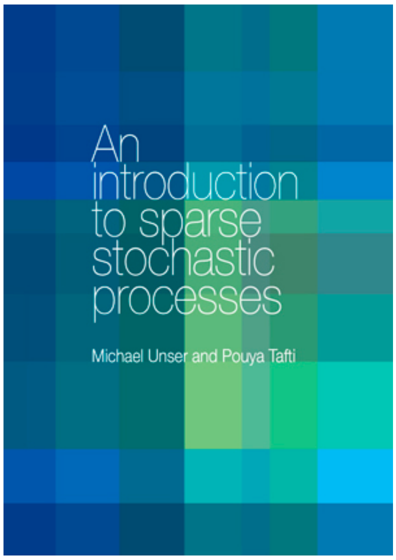

http://www.sparseprocesses.org